Drawing Triangles

Vocabulary

sufficient condition

included angle

▶ **BIG IDEA** Some combinations of measures of sides and angles are enough to determine all the other measures of sides and angles of the triangle.

Mental Math

Two angles of a triangle have the given measures. What is the measure of the third angle?

a. 3, 4

b. x, y
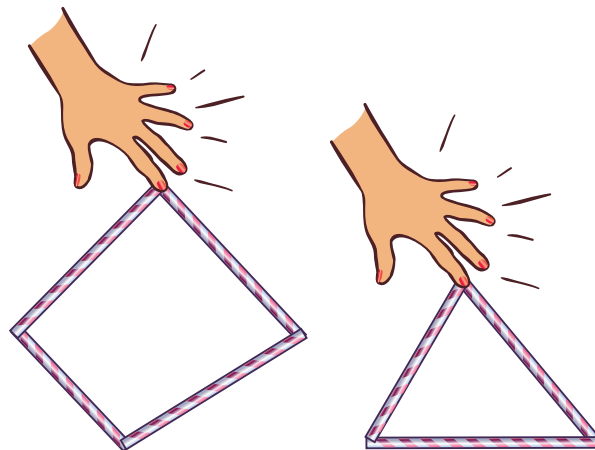
c. $75 + a, 75 - a$

The roofs of many houses are supported by a set of congruent triangular braces known as *trusses*. Supports, like trusses, usually involve triangles because triangles are *rigid*. To examine what this means, try the following activity.

Activity 1

MATERIALS straws, fasteners

Fasten four straws of different lengths together at their ends to create a quadrilateral. Do the same thing with three straws of different lengths. Play with the two figures you have created.

1. What do you think is meant by the statement that triangles are *rigid*?

2. How could you add an additional straw to your quadrilateral to make it rigid?

3. How does your answer to Question 2 relate to your answer from Question 1?

Exploring Sufficient Conditions for Triangles

Suppose you were in charge of setting the trusses for a house and, after setting up all of the trusses you have, you discover that you are one truss short. You now have to call the lumber yard to order one more. How could you ensure that you receive a triangular truss that is congruent to the rest? If you provide the lengths of the three sides and measures of the three angles, must you have them in a particular order? Could you tell less information about the triangle and still ensure that you receive a congruent triangle? Could you just tell the lumberyard the lengths of the three sides? What about the lengths of two sides and the measure of one angle? What about just providing the measures of three angles?

Whenever a set of conditions is enough so that all figures created will be congruent, then we call the set a **sufficient condition** for determining the figure. One way to study whether something is a sufficient condition is to draw several triangles satisfying that condition and see if they are all congruent.

Drawing a Triangle Given Three Sides

In Activity 1 you chose three lengths for the sides of a triangle, and then saw this triangle was rigid. Activity 2 examines whether one set of three lengths is a sufficient condition for determining the triangle.

Activity 2

MATERIALS DGS

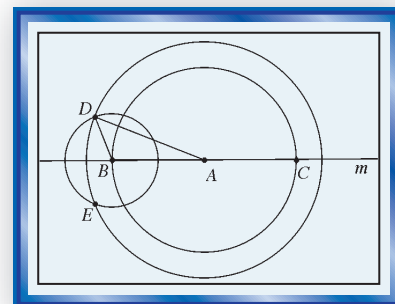**Using a DGS, draw triangles with sides 5 cm, 2 cm, and 6 cm.**

Step 1 Draw any line and call it *m*. Choose point *A* on *m*.

Step 2 Draw $\odot(A_1, 5 \text{ cm})$. ($\odot A_1$ identifies the first circle with center *A*.) Name $\odot A_1 \cap m = \{B, C\}$.
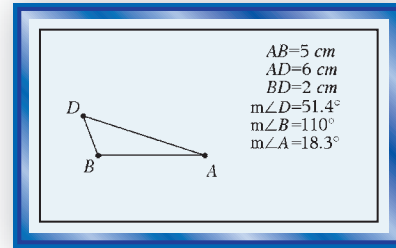
Step 3 Draw $\odot(A_2, 6 \text{ cm})$. ($\odot A_2$ identifies the second circle with center *A*.)

Step 4 a. Draw $\odot(B, 2 \text{ cm})$. Name $\odot A_2 \cap \odot B = \{D, E\}$.

  b. Draw $\overline{AD}$ and $\overline{BD}$. Your drawing should be similar to the one at the right.

  c. To the nearest tenth, what are the lengths of the sides of $\triangle ABD$?

Step 5 a. Measure the angles of $\triangle ABD$ to the nearest degree.

  b. In Steps 2 and 4 you made choices as to which points to call *B* and *D*. Would the measures of the angles of $\triangle ABD$ be different if you had named different points? Why or why not?

Here is what a DGS screen could look like after completing the steps of Activity 2, hiding the circles, and asking for lengths of sides and measures of angles in $\triangle ABD$. Do the angle measures in your triangles agree with those given at the right? If so, you might conjecture that the lengths of three segments is a sufficient condition for a triangle. This condition is called the SSS condition because it gives the lengths of the three sides.

$AB=5\ cm$
$AD=6\ cm$
$BD=2\ cm$
$m\angle D=51.4°$
$m\angle B=110°$
$m\angle A=18.3°$

Drawing a Triangle Given Three Angles

All rectangles have four 90° angles, but not all rectangles are congruent. Is the situation different for triangles? Is knowing the measures of the three angles in a triangle (AAA) a sufficient condition for congruence? This is the subject of Activity 3.
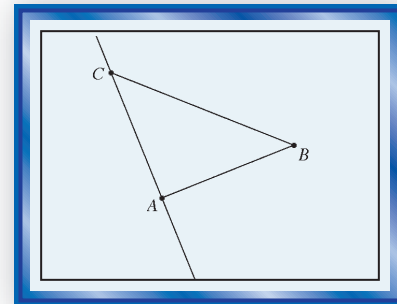
Activity 3

MATERIALS DGS

Step 1 Construct $\overline{AB}$.

Step 2 Construct line ℓ so that A lies on ℓ and $\ell \perp \overline{AB}$.

Step 3 Construct C on ℓ. Draw $\triangle ABC$ and measure its angles.

Step 4 Move C around until $m\angle C = 50$. What must $m\angle B$ be in this case?

Compare the triangle you drew to those of some of your classmates. Do they appear to be congruent? Do you think AAA is a sufficient condition?

Drawing a Triangle Given Two Sides and an Included Angle

Two sides of a triangle form part of an angle. That angle is said to be *included* by the sides, and is called an **included angle.** In Activity 4 you are asked to draw a triangle given the lengths of two sides and the measure of their included angle. This is called the SAS condition. Notice that the A is between the two Ss to indicate that the angle is included by the sides.
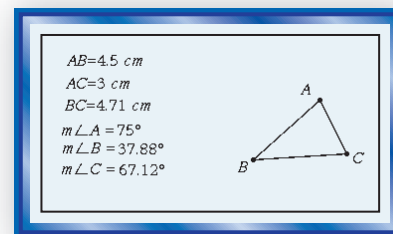
Activity 4

MATERIALS DGS

In a triangular sail $\triangle ABC$, $m\angle A = 75°$, $AB = 4.5$ m, and $AC = 3$ m. Make a scale drawing letting 1 cm in the drawing equal 1 meter of the actual sail.

a. Place $\triangle ABC$ and its measurements on the screen as in Activities 2 and 3.

b. Move point *A* so *AB* = 4.5 cm (within 0.02 cm).

c. Move point *C* so that m∠*A* = 75°.

d. Move point *C* along $\overline{AC}$ until *AC* = 3 cm, while keeping m∠*A* = 75°.

e. Record m∠*B*, m∠*C*, and *BC*.

f. Do the measurements in Part e agree (within 2° and 0.1 cm) with the measurements on the screen at the right?

AB=4.5 cm
AC=3 cm
BC=4.71 cm
m∠A =75°
m∠B =37.88°
m∠C = 67.12°

In the Activities in this lesson you were asked to draw a triangle given measures of some of its sides or angles. You then were asked whether SSS, AAA, or SAS was a sufficient condition to determine a unique triangle.

Other combinations of angles and sides are possible. Among these are ASA (two angles and the included side) and AAS (two angles and the nonincluded side). In the Questions you are asked to draw triangles given AA, SA, and SS. These questions ask whether you think everyone else's drawings will be congruent to yours. In other words, are these given conditions sufficient to determine a triangle?
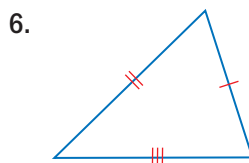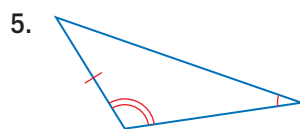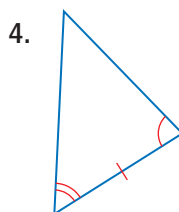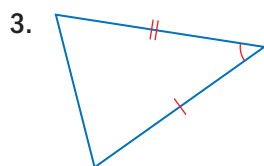
Activity	Given	Name for given condition
2	three sides	SSS
3	three angles	AAA
4	two sides and included angle	SAS

Questions

COVERING THE IDEAS

1. Give three reasons for studying triangles.

2. Explain what is meant by the statement, "triangles are rigid."

In 3−6, a triangle is given. Determine whether the information marked on the triangles is SSS, SAS, ASA, or AAS.

3.

4.

5.

6.

7. Draw a triangle with tick marks where the marked information gives the AAA condition.

8. Name two conditions you think are sufficient conditions for triangle congruence.

APPLYING THE MATHEMATICS

9. $\triangle ABC$ is isosceles with $AB = 5$ cm and $m\angle B = 48$. $\triangle DEF$ is isosceles with $DE = 5$ cm and $m\angle E = 48$. Explain why the triangles do not have to be congruent.

In 10–15, suppose a friend calls you on the phone and asks you for help on a geometry problem that involves a triangle $\triangle ABC$. You don't have the problem in front of you, so she tries to tell you how to draw the triangle.

a. Determine whether the set of instructions would allow you to construct $\triangle ABC$ so that your triangle looks exactly like your friend's triangle.

b. If all triangles you could construct from the given information are congruent, draw that triangle and state which one of the conditions in this lesson is met. If the given information allows for noncongruent triangles to be created, draw at least two noncongruent triangles satisfying the given information.
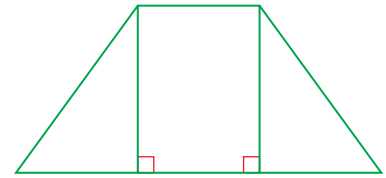
10. $m\angle A = 90$, $m\angle B = 28$

11. $AB = 3$ cm, $m\angle A = 57$

12. $AB = 10$ cm, $BC = 11$ cm, $AC = 9$ cm

13. $AC = 5$ cm, $BC = 2$ cm

14. $AB = 2.5"$, $BC = 1.5"$, $m\angle B = 30$

15. $m\angle A = 40$, $m\angle B = 60$

16. You are told that a certain desert island is located 50 miles from Abel Island and 40 miles from Bernoulli Island. Abel and Bernoulli Islands are 25 miles apart.

a. Draw a diagram showing that there are two possible locations for the desert island.

b. Does your answer contradict the statement that SSS is a sufficient condition for determining a triangle? Why or why not?

In 17–19, use a DGS as in Activities 2 and 3 to explore whether the given condition is sufficient to determine a triangle.

17. AAS   18. ASA   19. SSA

REVIEW

20. In the figure at the right, an isosceles trapezoidal region is cut up into a rectangle and two right triangles. Show how it could be cut up into four right triangles, and how it could be cut up into six right triangles. (**Lesson 6-6**)

21. Daniel walks one meter in a straight line, then turns 60° to his left. He repeats this five more times, for a total of six turns. At the end, is Daniel back where he started? Draw a picture of the route he took. (**Lesson 5-7**)

22. The numeral 8 can be broken into two congruent pieces in several ways. Draw two of these ways. (**Lesson 5-1**)

23. Suppose ℓ is a line and R is a rotation such that $R(\ell) \parallel \ell$. What can you say about the magnitude of R? (**Lesson 4-5**)

24. When driving, it is common wisdom to put your hands at 10 o'clock and 2 o'clock on the steering wheel. If you kept your hands firmly on the wheel, and rotated the wheel 90° to the left, at what hours would your hands be placed? (**Lesson 3-2**)

EXPLORATION

25. Find the congruent copy of each figure at the right in the picture below.

Source: Highlights for Children