Chapter 4

Lesson 4-4 Solving $ax + b = cx + d$

Vocabulary

general linear equation, $ax + b = cx + d$

BIG IDEA An equation of the form $ax + b = cx + d$ can be solved with just one more step than solving one of the form $ax + b = c$.

Solving $ax + b = cx + d$ with a CAS

In this lesson, you will see how to solve the **general linear equation, $ax + b = cx + d$.** First we explore solving them with a CAS.

Mental Math

Estimate to the nearest integer.

a. $4.1 \cdot 3.9$

b. $-6.2 \cdot 3.8$

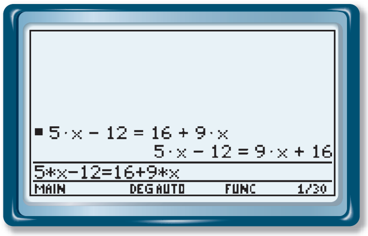
c. $-0.5 \cdot -5.05$

Example 1

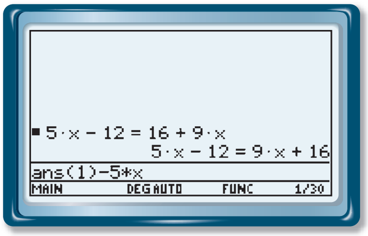
Solve $5x - 12 = 16 + 9x$ using a CAS.

Solution

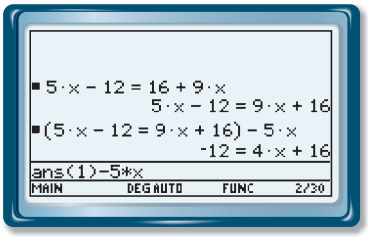
Step 1 Begin by entering the equation in a CAS.



Step 2 Each side has a variable term, $5x$ on the left and $9x$ on the right. Eliminate one of these by subtracting it from each side. We choose to subtract $5x$. On some CAS, you can just type in −5x. On other CAS, put the equation inside parentheses and then type −5x.

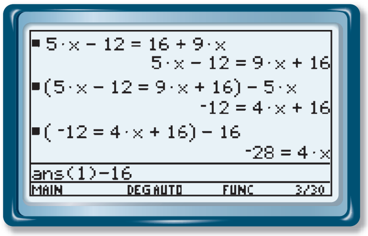


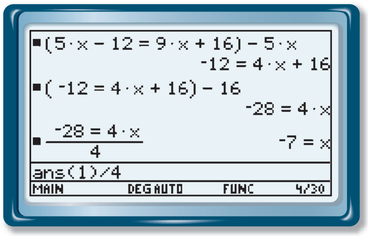
Step 3 Once you press ENTER, the CAS will subtract $5x$ from both sides of the equation. The result is $-12 = 4x + 16$. This is an equation of the type you solved in Chapter 3.



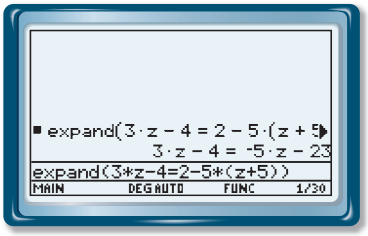
Step 4 Use the CAS to carry out the familiar steps to complete the solution. Subtract 16 from each side.

Step 5 Divide each side by 4. The equation is solved, $x = -7$.





As you can see, the methods you learned to solve simpler equations are also used to solve equations when the variable is on both sides. As before, sometimes the first step is to simplify one side of the equation. Some CAS will automatically simplify expressions. On other CAS, you must use the EXPAND command. The first step for one CAS is shown at the right. The original equation is $3z - 4 = 2 - 5(z + 5)$. The EXPAND command simplifies it to $3z - 4 = -5z - 23$. The next step would be to eliminate one of the variable terms, either $3z$ or $-5z$.



Activity

Use a CAS to solve the following equations. Remember, a CAS can help you make good decisions on the process of solving equations. Write down what you do to both sides of the equation as you go through the process.

1. $3z - 4 = 5z - 23$
2. $21 - 6x = 22 - -8x$
3. $-p = 13p - 42$
4. $33n - 102 + 4n = -23n - 252$
5. $9(9 - k) = 6k - 1$
6. $\frac{2}{3}b - \frac{6}{7} = \frac{5}{9}b + \frac{11}{3}$
7. $\frac{3}{4}(24 - 8y) = 2(5y + 1)$
8. $12.3 - (3.4w - 4.5) = 5.6(6.7 - 7.8w) - 8.9$
9. Write a description of the process you can use to solve $4x + 3 = 6x - 8$.
10. Use the process described in Question 9 to solve $4x + 3 = 6x - z$ for x without a CAS. Then check your solution with a CAS.
11. Solve $4x + t = 6x - z$ for x without a CAS. Then check your solution using a CAS.

Solving $ax + b = cx + d$ with Algebraic Processes

An important problem-solving strategy in mathematics and other fields is to turn a problem into a simpler one that you already know how to solve. In the Activity, you saw that to solve an equation in which the unknown is on both sides, you should turn it into an equation that has the variable on just *one* side.

Example 2

Solve $16x - 5 = 10x + 19$.

Solution Each side has a variable term, $16x$ and $10x$. Subtract one of these from each side to eliminate it. We choose to subtract $10x$.

$16x - 5 = 10x + 19$	Write the equation.
$16x - 5 - 10x = 10x + 19 - 10x$	Subtract $10x$ from each side.
$6x - 5 = 19$	Combine like terms.

The result is a simpler equation that you know how to solve.

$6x - 5 + 5 = 19 + 5$	Add 5 to each side.
$6x = 24$	Simplify.
$\frac{6x}{6} = \frac{24}{6}$	Divide each side by 6.
$x = 4$	Simplify.

Check Substitute 4 for x in the original equation.

Does $16(4) - 5 = 10(4) + 19$?
$64 - 5 = 40 + 19$
$59 = 59$ Yes.

But a question remains. In the first step, should you subtract $10x$ or $16x$ first? In solving $16x - 5 = 10x + 19$, we subtracted $10x$. We could have subtracted $16x$ instead. Either way works.

STOP QY

QY

Solve $16x - 5 = 10x + 19$ by first subtracting $16x$ from both sides.

Equations that Require Simplifying First

Now consider an equation that is more complicated. Again, we work to turn this problem into a simpler one.

GUIDED

Example 3

Solve $7 - (x + 5) = 4(x + 11)$.

Solution First, simplify each side. On the left side rewrite the subtraction in terms of addition.

$7 + -(x + 5) = 4(x + 11)$	Rewrite the equation.
$7 + \underline{\quad ? \quad} = \underline{\quad ? \quad}$	Distributive Property to remove parentheses
$-x + 2 = 4x + 44$	Simplify.

Now the equation has the form $ax + b = cx + d$. It is similar to Examples 1 and 2.

$-x + 2 + x = 4x + 44 + x$		Add x to both sides.
$\underline{\ ?\ }$	$= \underline{\ ?\ }$	Add like terms.
$\underline{\ ?\ }$	$= \underline{\ ?\ }$	Subtract 44 from each side.
$\underline{\ ?\ }$	$= x$	Divide each side by 5.

Check

$7 - (\underline{\ ?\ } + 5) = 4\ (\underline{\ ?\ } + 11)$ Substitute −8.4 for x in the original equation.

$7 - \underline{\ ?\ } = 4(\underline{\ ?\ })$ Remember to use the order of operations.

$10.4 = 10.4$ Yes, it checks.

Example 4

A dog breeder raises two kinds of dogs. At birth, the average puppy of breed A weighs 14.8 ounces and gains weight at a rate of 0.5 ounce per week. Breed B puppies are smaller at birth, weighing about 11.6 ounces. But they gain weight faster, at 0.9 ounce per week. How many weeks will it be before the puppies are the same weight?

The American Kennel Club (AKC) officially recognizes more than 150 breeds of dogs.

Source: American Kennel Club

Solution Let w = number of weeks that have passed. The weight of a breed A puppy will be $14.8 + 0.5w$. The weight of a breed B puppy will be $11.6 + 0.9w$. Set the expressions equal to each other to indicate that the weights are the same.

$14.8 + 0.5w = 11.6 + 0.9w$	Write the equation.
$14.8 + 0.5w - 0.5w = 11.6 + 0.9w - 0.5w$	Subtract $0.5w$ from each side.
$14.8 = 11.6 + 0.4w$	Add like terms.
$14.8 - 11.6 = 11.6 + 0.4w - 11.6$	Subtract 11.6 from each side.
$3.2 = 0.4w$	Simplify.
$\frac{3.2}{0.4} = \frac{0.4w}{0.4}$	Divide each side by 0.4.
$8 = w$	Simplify.

After 8 weeks, the weights of breed A and breed B puppies will be the same.

Check 1 Substitute 8 for w in the original equation.

Does $14.8 + 0.5(8) = 11.6 + 0.9(8)$?

$14.8 + 4.0 = 11.6 + 7.2$

$18.8 = 18.8$ Yes, it checks.

Check 2 Use a table and graph. Let x = the number of weeks that have passed, let Y_1 = weight of a breed A puppy, and Y_2 = weight of a breed B puppy. Enter `Y1=14.8+0.5x` and `Y2=11.6+0.9x`.

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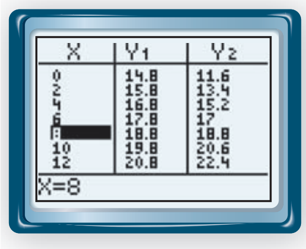
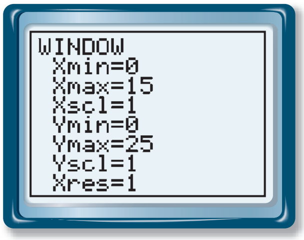
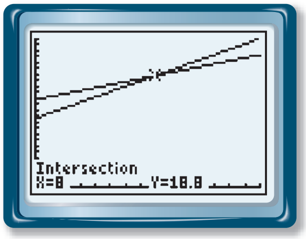
The row where $x = 8$ shows both breeds weigh 18.8 ounces. The INTERSECT command shows the two lines intersect at (8, 18.8).







No matter how long and complex the two sides of a linear equation are to begin with, the equation can never be more complicated than $ax + b = cx + d$ after each side is simplified.

Questions

COVERING THE IDEAS

1. The equation $9.3m - 4 = 11 + 2m$ is an equation of the form $ax + b = cx + d$. What are a, b, c, and d?

2. **Fill in the Blanks** An equation is solved below. Fill in the blanks to explain the steps of the solution.
   - a. $x - 4 = -3x - 7$ Add __?__ to each side.
   - b. $4x - 4 = -7$ Add __?__ to each side.
   - c. $4x = -3$ __?__ each side by __?__.
   - d. $x = -\frac{3}{4}$ Simplify.

3. a. To solve $10t + 5 = 4t + 7$, what can you add to both sides so t is on only one side of the equation?
   b. Solve the equation in Part a.

4. a. Solve $3x + 18 = 5x - 22$ by first adding $-5x$ to each side.
   b. Solve $3x + 18 = 5x - 22$ by first adding $-3x$ to each side.

5. Solve $250 + 0.01n = 70 + 0.03n$ by subtracting $0.03n$ from each side.

6. a. In solving $7m - \frac{23}{4} = -5m$, what advantage does adding $-7m$ to each side of the equations have over adding $5m$?
   b. Solve $7m - \frac{23}{4} = -5m$.

In 7–14, solve the equation and check your solution.

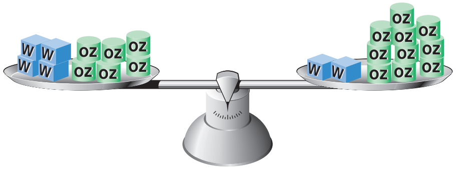
7. $4k + 39 = 7k + 6$
8. $9n = -6n + 5$
9. $14y + 5 = 8y - 1$
10. $46 - 8p = 19 + p$
11. $7 - m = 8 - 3m$
12. $1.55t - 2.85 = 8.4t + 10.85$
13. $3d + 4d + 5 = 6d + 7d + 8$
14. $4(x - 4) = 5(x - 3)$

15. Nebraska's population had been increasing at a rate of 13,300 people per year and reached 1,711,000 in 2000. West Virginia's population had been increasing at a rate of 1,400 people per year and reached 1,808,000 in 2000. If the rates of increase do not change in the future, when will the populations be equal?

16. The 2000 population of Dallas, Texas, was 1,189,000. It has been increasing at a rate of 21,500 people each year. The 2000 population of Philadelphia, Pennsylvania, was 1,518,000 and has been decreasing at a rate of 6,500 people each year. Assuming the rates do not change in the future, when will the populations of Dallas and Philadelphia be the same?

APPLYING THE MATHEMATICS

17. The boxes on the balance are of equal weight. Each cylinder represents 1 ounce.



a. What equation is represented by the balance?

b. Describe the steps you could use to find the weight of a box using a balance.

c. What is the weight of one box?

d. When you use a balance to represent solving $ax + b = cx + d$, why is there only one sensible choice of a variable term to eliminate from each side?

18. Five more than three times a number is three more than five times the number. What is the number?

19. In 2004, the women's Olympic winning time for the 100-meter freestyle in swimming was 53.84 seconds. The winning time had been decreasing at an average rate of 0.32 second per year. The men's winning time was 48.17 seconds and had been decreasing by an average of 0.18 second a year. Assume that these rates continue in the future.

a. What will the women's 100-meter winning time be x years after 2004?

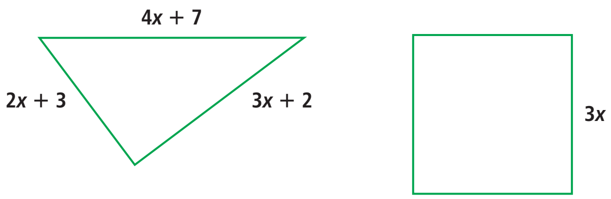
b. What will the men's 100-meter winning time be x years after 2004?

c. After how many years will the winning times be the same?

Jodie Henry of Australia won the gold medal in the 100-meter freestyle at the 2004 Olympic Games in Athens.

Source: International Olympic Committee

20. Refer to the figures below. The perimeter of the triangle is equal to the perimeter of the square. Find the length of a side of the square.

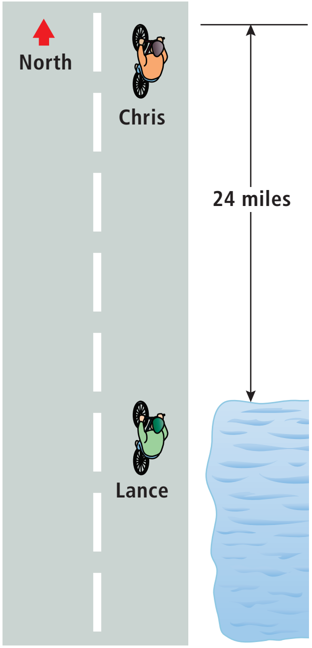


21. Lance and Chris are biking along the same road. Chris travels at a speed of 9 miles per hour, while Lance is faster and goes 13 miles per hour. Right now, Lance is next to a lake and Chris is 24 miles north of the lake.

a. Make a table showing how far each cyclist is from the lake at hours 0, 1, 2, 3, 4, 5, and 6.

b. Use the table to find how long it takes Lance to catch up with Chris.

c. Solve an equation to find how long it takes Lance to catch up to Chris.



(not drawn to scale)

REVIEW

22. The table at the right shows the cost of making copies of photos from negatives at two different photo shops. **(Lesson 4-3)**

a. For how many copies does it cost less to get photos copied at Ruby's than at Paula's?

b. For how many copies does it cost less to get photos copied at Paula's than at Ruby's?

Number of Copies	Ruby's Photos	Paula's Prints
1	\$0.75	\$0.80
2	\$1.25	\$1.40
3	\$2.00	\$2.00
4	\$2.75	\$2.60
5	\$3.50	\$3.20

23. Write an equation for the line containing the points $(8, d)$, $(0, d)$, and $(-4, d)$. **(Lesson 4-2)**

24. During the 2005–2006 National Basketball Association (NBA) season, Steve Nash scored approximately 17% of all the points that the Phoenix Suns scored, and Shawn Marion scored approximately $\frac{2}{10}$ of all the Suns' points. Nash and Marion combined to score 3,258 points that season. Approximately how many points did the team score in all? **(Lesson 3-8)**

25. Solve $9 - \frac{5c}{8} < \frac{21}{4}$. **(Lessons 3-8, 3-7, 3-6)**

26. Alvin wants to spend no more than \$18 at the state fair for admission and rides. If admission to the fair is \$8 and each ride costs \$1.25, how many rides can Alvin take? **(Lesson 3-6)**

27. The reciprocal of 6 is added to the opposite of 6. What is the result? **(Lessons 2-7, 2-4)**

28. Rewrite $18(3 - x)$ using each property. **(Lessons 2-1, 1-2)**

a. Commutative Property of Addition

b. Commutative Property of Multiplication

c. Distributive Property

In 29 and 30, estimate in your head to the nearest whole number. (Previous Course)

29. $\frac{9}{10} + \frac{12}{13} + \frac{8}{7}$

30. $9.92 \cdot 23$

EXPLORATION

31. This puzzle is from the book *Cyclopedia of Puzzles,* written by Sam Loyd, Jr. in 1914. If a bottle and a glass balance a pitcher, a bottle balances a glass and a plate, and two pitchers balance three plates, how many glasses will balance with a bottle?

QY ANSWER

$16x - 5 - 16x = 10x + 19 - 16x$

$-5 = 19 - 6x$

$-5 - 19 = 19 - 6x - 19$

$-24 = -6x$

$\frac{-24}{-6} = \frac{-6x}{-6}$

$4 = x$