Domain and Range 2

TEACHER NOTES

MATH NSPIRED

ENHANCED FOR THE ESSENTIAL MATH TEACHING PRACTICES

These Teacher Notes were enhanced to demonstrate how to integrate the Eight Essential Mathematics Teaching Practices into a lesson, with specific examples for when and how to address students' questions and work in class. Look for the Math Teaching Practices, which have been noted in italics, throughout the lesson to encourage student thinking and create effective mathematical discourse.

Math Objectives and Learning Goals

Establish mathematics learning goals to focus learning. *Prior to doing this lesson with students, you should establish the math objectives – which state what students will do as a part of the lesson – and the learning goals –which state what the students will understand as part of the lesson. Knowing those goals and objectives will help you focus your questions and guide discussions with students.*

- ***Learning Goal:*** *Students will understand what the domain and range of a function or relation is, and that they refer to the set of all possible values at which the function or relation is defined. By looking at the graph of a function or relation, students should understand that the domain refers to the x-coordinates and the range to the y-coordinates for every point in the function or relation. As such, they will understand that a function assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).*
- **Math Objective:** Students will be able to identify the domain and range for continuous functions or relations or discrete sets of points. Students will be able to identify the domain and range of a relation from its graph. Students will be able to write symbolic expressions to describe the domain and range of a relation.
- **Learning Goal:** *Students will recognize that different relations can have the same domain or the same range.*
- **Math Objective:** Students will create a graph with a given domain and range.
- ***Learning Goal:*** *Students should recognize situations in which the use of technology is appropriate in supporting their learning of a concept. They should understand the capabilities and limitations of their tools, so that they can reason appropriately when using them.*

Principles to Actions:

This lesson includes a guide to using the Essential Mathematics Teaching Practices, as described in *Principles to Actions: Mathematics Success for All.*

Tech Tips:

- This activity includes screen captures taken from the TI-Nspire™ CX handheld. It is also appropriate for use with the TI-Nspire™ family of products including TI-Nspire™ software and TI-Nspire™ App for iPad®. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

- **Math Objective:** Students will use appropriate tools strategically. (CCSS Mathematical Practice)

Vocabulary

- domain
- range
- infinity
- discrete
- continuous
- relation
- function
- interval notation

Lesson Materials:

Student Activity

- Domain_and_Range_2_Student.pdf
- Domain_and_Range_2_Student.doc

TI-Nspire™ document

Domain_and_Range_2.tns

About the Lesson

This lesson involves finding domain and range by changing the points on a graph. As a result, students will:

- Drag point *P* along the points in a scatter plot. From the ordered pairs, they will determine the domain and range.
- Determine the domain and range of several different relations by dragging a point along the graph.
- Use interval notation and inequalities to describe the domain and range.
- Compare domains and ranges that are discrete with those that are continuous over an interval.
- Sketch graphs given a domain and range.

Implement tasks that promote reasoning and problem solving: *This lesson should be included to introduce or expand the concepts of domain and range for discrete or discontinuous functions and relations. Depending on the prior knowledge of students, parts of this lesson can be used with introductory-level students, such as using lines and sets of points. It is recommended that students explore the concepts in this activity after they have done a basic introduction to domain and range. Allowing students the time to visualize, think about and discuss the concepts in this activity helps them to develop a deeper meaning of the mathematics.*

Students may work independently or in small groups for this lesson. It is important for the teacher to monitor students' work, to ask questions throughout the lesson and to include class discussion in order to ensure student understanding.

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- Send out the *Domain_and_Range_2.tns* file.
- Monitor student progress using Class Capture.
- Use Live Presenter to spotlight student answers.

Activity Materials

- Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software

What's Next?

Follow-up to this lesson may include the following activities from Math Nspired, such as Graphic Designing with Transformed Functions, in which students create a picture using graphs of piecewise functions.

Discussion Points and Possible Answers

TI-Nspire™ Navigator™ Opportunity: *Class Capture* or *Live Presenter*

See Note 1 at the end of this lesson.

Tech Tip: If students experience difficulty dragging a point, check to make sure that they have moved the cursor (arrow) until it becomes a hand (÷) getting ready to grab the point. Also, be sure that the word *point* appears. Then press / x to grab point *P* and close the hand ({). When finished moving the point, press d to release the point.

Teacher Tip: Students should not begin dragging point *P* until they have recorded the answer to part **a** in question 1. If they do, the can click on the reset slider and then drag the point.

For pages 2.1–5.1, there is a reset slider to remove the tracing of the domain and range.

Elicit and use evidence of student thinking: *Prior to starting this activity, assess your students' understanding of representing intervals symbolically and graphically:*

- *Have students draw the inequality, -3< x ≤ 5, on a number line. Ask the students to explain how they know what the graph looks like, given the notation. As the students discuss this inequality, be sure that they note that x can be any value between -3 and 5; not including -3 but including 5.*
- *On a coordinate grid, have the students graph y > -3, then describe the domain and range. Repeat for y ≤ 5; for x>-3; and for x≤ 5. Ask the students to compare the graphs, domain and range for each inequality. Think about the possible points represented by these intervals.*

Be sure that students have an opportunity to understand domain and range both graphically and symbolically for this lesson.

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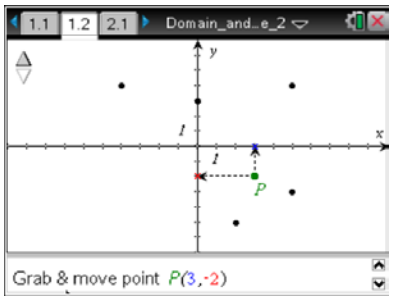


1. Grab and move point *P* to each point on the scatter plot and note the changes.

   a. Name the ordered pair for point *P*. What is the relationship between this ordered pair and the cross mark on each axis?

Answer: Answers may vary for the ordered pair for point *P*. The cross mark on the *x*-axis is at the *x*-coordinate of the ordered pair. If using the TI-Nspire™ CX handheld or the TI-Nspire™ App for iPad, it is marked in blue. The cross mark on the *y*-axis is at the *y*-coordinate of the ordered pair. If using the TI-Nspire™ CX handheld or the TI-Nspire™ App for iPad, it is marked in red.

   b. Grab and drag point *P* to each of the circles on the scatter plot. As you move from point to point, record the coordinates in the table.

Answer: The completed table is below.

x	*y*
–4	4
0	3
2	–5
3	–2
5	–3
5	4

Teacher Tip: The order in which students place the ordered pairs in the table does not matter.

c. State the domain and range of the relation.

Answer: Domain: {–4, 0, 2, 3, 5}; Range: {–5, –3, –2, 3, 4}

Teacher Tip: It is easier to specify the set if the domain and range are listed from least to greatest, but order is not critical. The values should not be repeated.

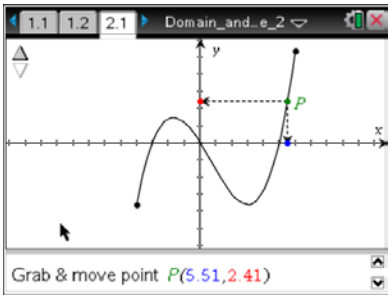
d. Where on the graph do you see the domain represented? The range represented?

Answer: The domain of the graph can be seen as the cross marks on the *x*-axis or as the highlighted blue section on the *x*-axis. The range can be seen as the cross marks on the *y*-axis or as the highlighted red section on the *y*-axis.

Move to page 2.1.

2. Move point *P* back and forth along the entire graph.

a. What does the highlighted portion along the *x*-axis represent?



Answer: The highlighted portion along the *x*-axis represents the domain of the graph. If you are using the TI-Nspire™ CX handheld or the TI-Nspire™ App for iPad, the highlighted portion will be blue.

b. What does the highlighted portion along the *y*-axis represent?

Answer: The highlighted portion along the *y*-axis represents the range of the graph. If you are using the TI-Nspire™ CX or the Nspire™ App for iPad, the highlighted portion will be red.

Facilitate meaningful mathematical discourse. *It is important to allow your students to discuss this problem to give them an opportunity to analyze and compare the ideas of others. As you listen to students work together, think about how you want to sequence students' answers for explanation to the whole class. The order in which these questions are posed is important to helping students build their understanding of the concepts. Be sure that students have time to internalize each response before moving on to the next point.*

Practices for effective discourse	***Teacher response/ actions***
Anticipating student responses	*Prior to the lesson, think about the types of answers your students may give and how you may respond. It is easier to approach the lesson with a plan for the discussion than attempting to do it on the spot.*
Monitoring student work	*As students work through the problem, make some observations about student work. Rather than providing a direct path to an answer, allow students time to reason on their own or to discuss possibilities with another student. If a student is stuck, you may ask questions to help them get started.*
Selecting students to present work	*As you observe students' work, identify solutions that can be presented to the class or group. Ask students to present their ideas in the following order:*
Sequencing students' responses	*Select student solutions to help build ideas:* *1. Solution to part c* *2. Solution to part d* *3. Solution to part e* *A solution with a misconception may be useful as well.*
Connecting students' responses to key ideas	*After a short discussion, have students work in pairs or small groups to discuss part f in this problem.*

c. How do the domain and range in this problem differ from the domain and range in question 1?

Answer: The domain and range in this graph are continuous over an interval, whereas the domain and range in question 1 are discrete.

d. State the domain as an inequality and in interval notation.

Answer: Domain: $-4 \le x \le 6$; $x \in [-4, 6]$

Teacher Tip: There may be some confusion between interval notation and an ordered pair. To alleviate this confusion, you may want to use "$x \in$," which means "x is an element of," before the interval notation.

e. State the range as an inequality and in interval notation.

Answer: Range: $-4 \le y \le 5$; $y \in [-4, 5]$

Teacher Tip: There may be some confusion between interval notation and an ordered pair. To alleviate this confusion, you may want to use "$y \in$," which means "y is an element of," before the interval notation.

f. If the endpoints of the graph were open circles, how would the domain and range change? State the new domain and range as inequalities and in interval notation.

Answer: The domain would be between –4 and 6, not including –4 and 6. The range would be between –4 and 5, not including –4 and 5.

Domain: $-4 < x < 6$ Range: $-4 < y < 5$

$x \in (-4, 6)$ $y \in (-4, 5)$

Teacher Tip: Emphasize what open and closed circles mean on a graph, in words, and in an inequality. The endpoint may or may not be part of the domain and range. In this problem, if the circles were open, the inequality symbols would be strictly less than, *not* less than or equal to.

Use and connect mathematical representations: *As students complete the first two problems, be sure that they understand the connection between the different representations of the domain and range.*

Strategy	***Example***
Encourage purposeful selection of representations.	*Describe the domain and range of this relation using a table, graph, symbols or verbal forms.* *If you can't use a certain representation, explain why.*
Engage in dialogue about explicit connections among representations.	*How would you describe the domain and range in this problem using interval notation?* *How would you describe to a friend the domain and range of the graphs in Questions 1 and 2? You can use any representation to describe.*
Alternate the direction of the connections made among representations.	*How could you describe the domain and range for a relation using a table?* *When would this not work? Why?*

Pose purposeful questions: *Follow the students' discussions from the previous problem with questions that help solidify the concepts. Be sure to ask questions as they relate to student work. Be mindful of the questions you ask so that they do not lead directly to an answer. Allow students time to think and reason. Their responses will help identify the route you should take with the next question.*

Teacher Question	***Question Type***
When would it be possible for the endpoints in the interval to not be included in the domain or range?	*Probing student thinking*
What can you conclude about the relationship between the domain shown on the graph and the domain written in interval notation?	*Making the mathematics visible*
Explain the meaning of the open and closed circles on a graph, in words and in an inequality.	*Making the mathematics visible*
How would you know that a relation is a function?	*Encouraging reflection and justification*

How would you know if the relation is continuous?	*Encouraging reflection and justification*
What would happen if the point at (-4,-4) was open instead of closed?	*Encouraging reflection and justification*

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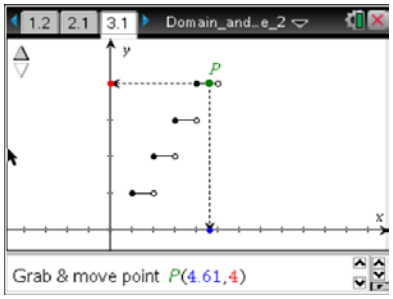

3. Move point *P* back and forth along the entire graph.

   a. State the domain and range of the graph.
   Answer: Domain: $1 \leq x < 5$ or $x \in [1, 5)$

   Range: {1, 2, 3, 4}

Teacher Tip: As point *P* is being dragged, make sure that the hand remains close to point *P*. If the hand moves too far away from point *P*, the students will not be able to see the entire domain. Point out all of the representations that are being seen.

   b. How does the domain of this graph compare to the domain of the graph in question 1? Question 2?

   Answer: The domain of this graph is over an interval (continuous) with two defined endpoints, whereas the domain in question 1 is discrete. The domain in question 2 is like the domain in this graph. It is over an interval with two defined endpoints. However, both endpoints are included in the domain in question 2.

   c. How does the range of this graph compare to the range of the graph in question 1? Question 2?

   Answer: The range of this graph is discrete, like the range in question 1. The range in question 2 is over an interval (continuous) with two defined endpoints.

Pose purposeful questions. *Students can be asked to consider:*

- *Why is it important to notice whether the endpoints of an interval are in the domain?*
- *What strategies can you use to find the domain and range of a function?*
- *Can you find a function whose range consists of a single real number?*
- *What would happen to the function in Question 3 if the point at (2,1) was filled in?*

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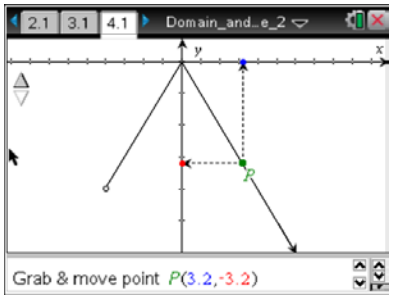

4. Move point *P* back and forth along the entire graph.
   a. State the domain and range as inequalities and in interval notation.

      Answer: Domain: $x > -4$ or $x \in (-4, \infty)$

      Range: $y \leq 0$ or $y \in (-\infty, 0]$

Support productive struggle in learning mathematics. *It is important to ask questions and to help make mathematical connections. However, it is just as important to allow students some time to grapple with the mathematics. Allowing time for students to work alone in small groups will allow them time to internalize concepts being discussed in class. Be mindful of the time you give students to respond, of the time between the subsequent questions you ask and of the amount of "help" you provide students as they form ideas about the concepts.*

This problem might be challenging for students for any of the following reasons.

- *The left-most endpoint is open. Allow students time to discuss part b in their pairs or small groups. Ask students to compare the endpoints in the graphs of the previous problems to the endpoints here. Ask students to explain how to write the interval for the domain when the graph is given.*
- *Some students may say that the range is all real numbers; they fail to note the maximum point at y=0. Ask students to justify the meaning of "all real numbers"; and explain whether the range here includes those numbers.*
- *Some students may struggle with putting the open end of the interval into notation. Have students think about what $-\infty$ might mean in terms of the range and what ∞ might mean in terms of the domain. Show students examples of graphs that have a domain that is all positive real numbers and help students write the domain in interval notation.*

   b. What is different about the domain and range of this graph compared to the others?

      Answer: The domain of this graph is over an interval (continuous) with one defined endpoint. The graph continues infinitely in the positive direction, whereas the domain in question 1 is discrete. The domain in question 2 is like the domain in this graph. It is over an interval. However, the graph in question 2 has two defined endpoints.

      The range of this graph is over an interval (continuous) with one defined endpoint. The graph continues infinitely in the negative direction, whereas the range in question 1 is discrete. The range in question 2 is like the range in this graph. It is over an interval. However, the graph in question 2 has two defined endpoints.

Use and connect mathematical representations: *As students complete this problem, be sure that they understand the connection between the different representations of the domain and range. The table below highlights some strategies for helping students connect the mathematical representations.*

Strategy	***Example***
Encourage purposeful selection of representations.	*Describe the domain and range of this relation in table, graphic, symbolic and verbal forms, when appropriate.* *Would a table be an appropriate representation in this situation; why or why not?*
Engage in dialogue about explicit connections among representations.	*How would you describe the domain and range in this problem using interval notation?* *Be sure that students know how to use parentheses and brackets to denote open and closed intervals.* *On the graphs in the previous problems, there have been arrows, open circles, and closed circles at the endpoints. How do these affect the domain and range?*
Alternate the direction of the connections made among representations.	*Given the domain and range interval notation, how would you explain to someone how you could draw a graph?* *Would this graph necessarily be the only graph related to this domain and range?*

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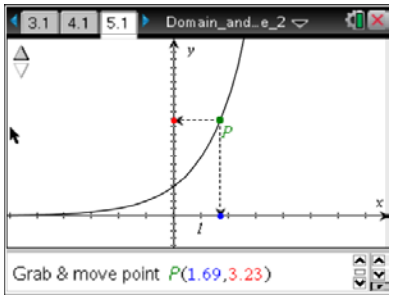

5. Move point *P* back and forth along the entire graph.
   a. State the domain and range as inequalities and in interval notation

      Answer: Domain: $-\infty < x < \infty$ or $x \in (-\infty, \infty)$

      Range: $y > 0$ or $y \in (0, \infty)$

b. What is different about the domain and range of this graph compared to the others?

Answer: The domain of this graph is the set of all real numbers. The graph continues infinitely in both directions, whereas the domain in question 1 is discrete. The domain in question 2 is like the domain in this graph. It is over an interval. However, the graph in question 2 has two defined endpoints.

The range of this graph is the set of all real numbers greater than 0. The graph continues infinitely in the positive direction, whereas the range in question 1 is discrete. The domain in question 2 is like the range in this graph. It is over an interval. However, the graph in question 2 has two defined endpoints.

Use and connect mathematical representations. *The next problem will help students make the connection between the symbolic and graphic representations. As students create new segments, be sure that they are stating or writing the new domain and range. Allow students to work together to discuss the new graphs and their corresponding domain and range.*

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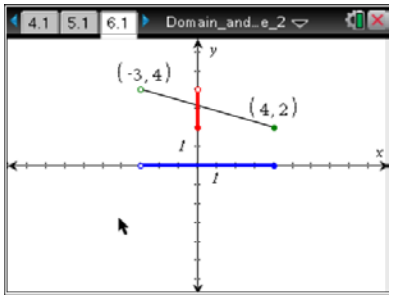

6. Grab and move the endpoints of the line segment to new locations.

   a. What do you notice about the domain and range as you drag the endpoints?

   Answer: As you move the endpoints of the segment, the domain and range change as well to reflect the new domain and range.

TI-Nspire™ Navigator™ Opportunity: *Class Capture* or *Live Presenter*

See Note 2 at the end of this lesson.

Elicit evidence of student understanding. *For part b, allow students to compare their graphs with those of other students. Have students consider - Is it possible for two graphs using this model to have the same domain and range, but look different? Is it possible for any two relations to have the same domain and Can you make move the segment to make one graph that has the same range and domain? How? Then, follow up with How does what you just did allow you to create more (different) graphs that have the same domain and range as the graph in this example?*

b. Move the open endpoint to (–1, 4) and the closed endpoint to (2, –3). State the domain

and range as inequalities and in interval notation.

Answer: Domain: $-1 < x \leq 2$ or $x \in (-1, 2]$

Range: $-3 \leq y < 4$ or $y \in [-3, 4)$

Use and connect mathematical representations: *As students complete this problem, be sure that they understand how to represent the closed and open ends of the interval.*

Strategy	***Example***
Encourage purposeful selection of representations.	*How would/could you describe the domain and range given a table or graph from this example?*
Engage in dialogue about explicit connections among representations.	*How would you describe the domain and range in this problem using interval notation?* *How would you verbally describe the domain and range?* *Describe the difference between the open circle and closed circle on the graph in terms of the interval notation.*
Alternate the direction of the connections made among representations.	*Allow students some time to talk about the graphs they just made and the domain and range of those graphs.*

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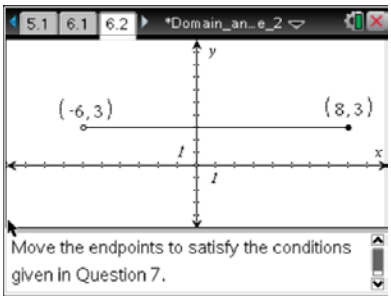


7. Grab and move the endpoints of the line segment to satisfy each of the following conditions:
   a. The open endpoint is (–4, –3) and the closed endpoint is (3, 4). State the domain and range using inequalities and interval notation

   Answer: Domain: $-4 < x \leq 3$ or $x \in (-4, 3]$

   Range: $-3 < y \leq 4$ or $y \in (-3, 4]$

TI-Nspire™ Navigator™ Opportunity: *Quick Poll*

See Note 3 at the end of this lesson.

b. The domain is [–7, 7) and the range is (–4, 5]. Write the domain and range as inequalities. State the endpoints of the line segment and indicate which endpoint is open.

Answer: Domain: $-7 \le x < 7$

Range: $-4 < y \le 5$

The endpoints are (–7, 5) and (7, –4).

The open endpoint is (7, –4).

Teacher Tip: Students may have difficulty naming the correct endpoints. You may need to redirect their attention to the brackets and parentheses in the domain and range to ensure the correct endpoints are named.

c. Given a domain of $-3 \le x < 8$ and a range of $y = -2$, state the endpoints of the line segment and indicate which endpoint is open.

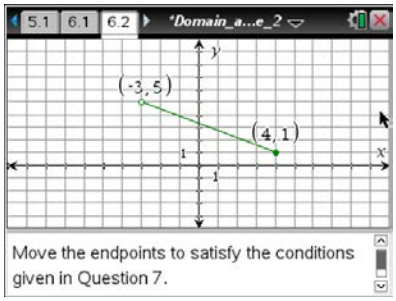
Answer: The endpoints are (–3, –2) and (8, –2). The open endpoint is (8, –2).

d. The domain is $x = 5$. State the endpoints of the line segment and indicate which endpoint is open. Compare your answers with another student's. Explain how you can have different answers and still both be correct.

Answer: Answers may vary, and either endpoint may be open. However, all ordered pairs should contain 5 as the *x*-coordinate.

8. Sketch a graph and write a description of a function or relation that satisfies the following conditions. Compare your answers with another student's. How are they alike or different?
   a. Domain: $-3 < x \le 4$; Range: $1 \le y < 5$

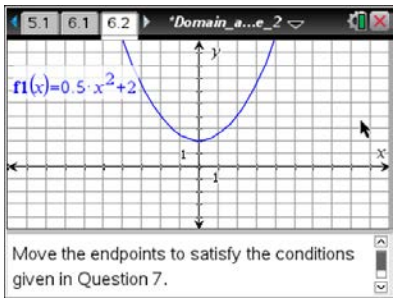
   Possible Answer: See example at right.



TI-Nspire™ Navigator™ Opportunity: *Use Class Capture to show the graphs students created to the class. Ask students to compare the domain and range of their graphs with others'. This should be done even if you don't have a TI-Nspire Navigator system.*

b. Domain: (–∞, ∞); Range: [2, ∞)

Possible Answer: See example at right.



Build procedual fluency from conceptual understanding. *This question will help students extend their knowledge of domain and range to applying it to other situations.*

Example 1: Andre needs to fill up his truck with gasoline to drive to and from school next week. If gas costs $2.79 per gallon, and his truck holds a maximum of 28 gallons, analyze the domain, range, and function values by answering the following.

a. Describe in words and in interval notation the domain and range of the situation.
b. Write an equation that models the total cost of the gas.
c. Describe how you would find the maximum Peter can spend on gas.

Example 2: Mari is sending a package to her grandmother. The shipping rate for small packages that weigh up to 2 lbs is $3.25. For packages that weigh bewteen 2 lbs and 5 lbs, the shipping rate is $4.55. For small packages from 5 lbs to 8 lbs, the postage is $6.75.

a. How would you describe the domain and range for this relation?
b. What would a graph of the relation look like?

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:

- How to determine the domain and range of a relation from its graph.
- The connection between graphical and numerical representations.
- How to read and write symbolic expressions to describe the domain and range of a relation.
- How to sketch a graph with a given domain and range.

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Note 1

Entire Document, *Class Capture* or *Live Presenter*: If students experience difficulty with dragging the point or understanding the results, use Class Capture to show the solution to the students. Highlight the bold areas on the axes that represent the domain and range.

Note 2

Question 6a, *Class Capture* or *Live Presenter*: Use this opportunity to show different students' graphs and point out how the domain and range are affected by changing the endpoints of the segment.

Note 3

Question 7a, *Quick Poll (Multiple Choice or Open Response)*: Use Quick Poll as an opportunity to assess students' understanding of domain and range for a function. Have students identify the domain and range for this function. As a follow-up question, ask students what kind of line will have a domain of $x = 2$.