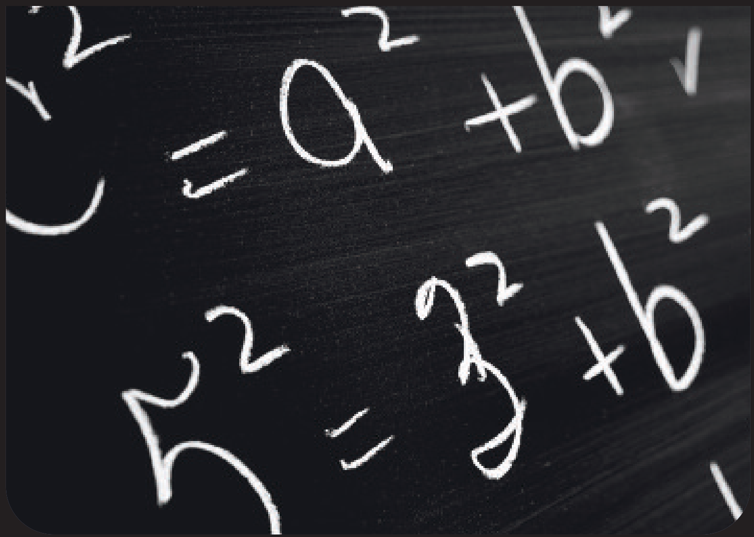
Why Isn't the Mathematics We Learned Good Enough for Today's Students?

The Imperative of Mathematics Literacy



Briefing Paper Series on Mathematical Literacy

Improving the mathematics skills of our citizenry has been a major concern for educators, policy makers, and the general public since long before Sputnik ushered in "new math." With the most recent decade of education reform and the advent of "new-new math," advances in mathematics research and education have led to both fruitful exchanges of ideas and challenging debates. Never before has it been so clear that mathematical literacy is vital for our nation's economic growth, security, and civic progress. And never has the call to bring *all* children to high levels of mathematical literacy been sounded so forcefully. Yet, though our culture, our country, and our schools by and large expect all adults to be able to read, we do *not* expect all adults to be proficient in mathematics. (How often does someone utter, "I was never good at math," only to be met with nods of understanding and compassion?) By and large, Americans accept the kinds of results that come from the widespread belief that not all children can learn mathematics beyond "arithmetic."

Believing that all children *can* learn mathematics, and, indeed, that they must, the Council of Chief State School Officers and Texas Instruments Incorporated, have joined together in a partnership to respond to the clarity of purpose and urgency of mission felt in the states today around mathematics education. This partnership will investigate the influences on mathematics education and develop recommendations for effective state actions to lead to improved student performance in mathematics. This paper is the introduction to a series of papers designed to analyze the imperatives and opportunities in several critical areas of mathematics education. The papers will explore the depth and type of mathematical knowledge that students will need to be successful in today's society; how that depth and type of mathematical knowledge is best taught and what this implies for schools and classrooms; and the conditions that need to be established to create this kind of teaching and learning in every classroom. Specific topics that will be addressed by this series include

- The Imperative of Mathematical Literacy
- Standards, Curriculum, Instruction, and Assessment
- Teacher Preparation and Professional Development
- Teacher Recruitment, Assignment, and Retention
- Opportunities for Support and Partnerships

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Why Isn't the Mathematics We Learned Good Enough for Today's Students? The Imperative of Mathematical Literacy

I always got A's in advanced math in high school, but I guess sometimes I didn't understand how everything worked. You know, once you figured out the calculations for problems that looked a certain way, you could just use those for all the problems like it. I got great grades, but I guess I never knew why. Now I'm flunking out of my college math course.

—a recent high school graduate

Students need to learn the basics! Now teachers have them playing with blocks and getting answers from the calculator. What are they learning from that?

—a parent

With the old textbook, I led students through the different kinds of problems and equations—just like in the book. With this new curriculum, there's group work and discussion, all kinds of questions come up—and there are a bunch of different possible approaches. I'm afraid of not knowing the answer. I don't know enough math to teach this way!

—a teacher

We need people who can figure out how to solve a problem; that's more than just knowing how to plug numbers into a calculator in the right order.

—an employer

Why do we have to pay for all of this professional development for teachers? Didn't their colleges teach them how to teach math?

—a policymaker

The American public is struggling to understand how schools can prepare students to succeed in the colleges, workplaces, and civic communities of the future. This paper describes why teachers must have effective methods for teaching mathematics and why schools, parents, and community partners ought to find ways to promote mathematics literacy for all. Key issues in the struggle are explored in greater detail in subsequent papers in this series.

Mathematical Literacy in a Changing Economy, Civil Society, and Information Age

We are educating the majority of American students in schools that closely resemble those that our fathers and mothers attended, but students today must succeed in a culture and an economy that has changed dramatically from that of a generation or two ago. At one time in our history, low-skill jobs fueled a strong economy and provided good wages to low-skill workers. This, however, is no longer the case; our 21st century economy demands higher skills from all workers. In a recent report, the American Diploma Project studied the entry-level requirements of jobs that pay enough to support a family, provide benefits, and provide pathways for career advancement—jobs that will represent 62 percent of all the jobs in our economy over the next ten years.[1] They found that entry-level workers in these sorts of jobs need to understand numbers and numerical operations and to have at least a working knowledge of algebra, geometry, data interpretation, probability and statistics. This is content that typically has been expected only of college-bound students. Indeed, when the American Diploma Project (ADP) compared their findings about employers' requirements to the expectations of two- and four-year state colleges and universities, they found that all students need similar levels of mathematics knowledge and skill whether they are college bound or they plan to go directly into the workforce.

This is because the *average* worker is now placed in situations where higher level skills—such as the ability to solve non-routine, multi-step problems (as Stanford University mathematics professor R. James Milgram puts it, "solving a problem where the answer is not immediate and requires a novel idea from the student.")[2]—are critical to their success. "Creative workers" who are able to analyze a situation, hypothesize about causes and effects, and craft new solutions will be in great demand in the coming years, will garner competitive wages, and will fuel a high performance economy.[3] High quality mathematics education can prepare workers with these skills and intuitions.

This level of mathematical knowledge is not only crucial for individual success, it is important for the future of our nation. Alan Greenspan, chairman of the Federal Reserve System, recently commented that the future of this country depends upon the skill of our people,[4] which translates into a dependence on our education system to teach children to higher levels of performance. Yet, though the United States has a history of producing a great many high skill knowledge workers, we are not keeping up with advances in other countries. Over the next fifteen years, 3.3 million jobs will move to East Asia—*not* because of cheap labor, but because other countries are educating more of their workers to perform in high skill environments. China is graduating two times as many students with BA degrees and almost four times as many engineering students as are graduating in the United States.[5] India graduated one million more students than the United States.[6]

Couple this with the fact that our middle and high school students do not fare well on international comparisons of performance and we have a situation that we cannot abide for long. The United States, along with 44 other nations, participated in the 2003 eighth grade Trends in International Mathematics and Science Study (TIMSS, formerly known as the Third International Mathematics and Science Study). Offered in 1995, 1999, and

The American Diploma Project surveyed employers and post-secondary faculty to determine the knowledge and skills high school graduates will need. They describe mathematical reasoning:

A Note About Mathematical Reasoning

The study of mathematics is an exercise in reasoning. Beyond acquiring procedural mathematical skills with their clear methods and boundaries, students need to master the more subjective skills of reading, interpreting, representing and "mathematicizing" a problem. As college students and employees, high school graduates will need to use mathematics in contexts quite different from the high school classroom. They will need to make judgments about what problem needs to be solved and, therefore, about which operations and procedures to apply. Woven throughout the four domains of mathematics—Number Sense and Numerical Operations; Algebra; Geometry; and Data Interpretation, Statistics and Probability—are the following mathematical reasoning skills:

- Using inductive and deductive reasoning to arrive at valid conclusions.
- Using multiple representations (literal, symbolic, graphic) to represent problems and solutions.
- Understanding the role of definitions, proofs and counter-examples in mathematical reasoning; constructing simple proofs.
- Using the special symbols of mathematics correctly and precisely.
- Recognizing when an estimate or approximation is more appropriate than an exact answer and understanding the limits on precision of approximations.
- Distinguishing relevant from irrelevant information, identifying missing information, and either finding what is needed or making appropriate estimates.
- Recognizing and using the process of mathematical modeling: recognizing and clarifying mathematical structures that are embedded in other contexts, formulating a problem in mathematical terms, using mathematical strategies to reach a solution, and interpreting the solution in the context of the original problem.
- When solving problems, thinking ahead about strategy, testing ideas with special cases, trying different approaches, checking for errors and reasonableness of solutions as a regular part of routine work, and devising independent ways to verify results.
- Shifting regularly between the specific and the general, using examples to understand general ideas, and extending specific results to more general cases to gain insight.

Source: American Diploma Project, (2004). *Ready or not: Creating a high school diploma that counts.* (Washington, DC: Achieve, Inc.). p. 55.

2003, TIMSS provides trend data on students' mathematics and science achievement from an international perspective. Of the 45 nations, U.S. eighth-graders in 2003 were essentially in the middle of the pack; they performed lower than their peers in 9 nations in mathematics, similarly to their peers in 10 nations, and outperformed their peers in 25 countries in mathematics.[7]

The good news from TIMSS trend data is that the United States is improving in its mathematics achievement. Yet the concern is that the comparative level of mathematics demonstrated on TIMSS does not suggest that the United States will sustain its historic predominance in the international economy.

The economic imperative is not the only reason to pay attention to mathematical literacy in the U.S. While the information age has ushered in new economies—complete with their own legal issues and workplace challenges—it has also brought about new pressures on citizens. Advances in technology and the widespread use of the internet and satellite and cable television mean that citizens are bombarded with information, often unfiltered and unanalyzed. In order to make competent social decisions, average citizens must be able to understand complex circumstances and make sense of a wide array of new facts and opinions. Understanding basic statistics and probability is a requirement in today's information rich age.

Figure 1: Average mathematics scale scores of eighth-grade students, by country 2003.

COUNTRY	AVERAGE SCORE
International Average*	466
Singapore	605
Korea, Republic of	589
Hong Kong SAR** †	586
Chinese Taipei	585
Japan	570
Belgium-Flemish	537
Netherlands**	536
Estonia	531
Hungary	529
Malaysia	508
Latvia	508
Russian Federation	508
Slovak Republic	508
Australia	505
(United States)	504
Lithuania‡	502
Sweden	499
Scotland**	498
Israel	496
New Zealand	494
Slovenia	493
Italy	484
Armenia	478
Serbia‡	477
Bulgaria	476
Romania	475
Norway	461
Moldova, Republic of	460
Cyprus	459
(Macedonia, Republic of)	435
Lebanon	433
Jordan	424
Iran, Islamic Republic of	411
Indonesia‡	411
Tunisia	410
Egypt	406
Bahrain	401
Palestinian National Authority	390
Chile	387
(Morocco)	387
Philippines	378
Botswana	366
Saudi Arabia	332
Ghana	276
South Africa	264

Average Higher Than US Average
Average Not Measurably Different From US
Average Lower Than US Average

DATA SOURCE: International Association for the Evaluation of Educational Achievement (IEA), *Trends in International Mathematics Study (TIMSS).* (2003).

SOURCE FOR CHART AND NOTATIONS: United States Department of Education, National Center for Education Statistics. (2005). *TIMSS 2003 results.*

* The international average reported here differs from that reported in Mullis et al. (2004) due to the deletion of England. In Mullis et al.,the reported international average is 467.

**Met international guidelines for participation rates in 2003 only after replacement schools were included.

† Hong Kong is a Special Administrative Region (SAR) of the People's Republic of China.

‡ National desired population does not cover all of the international desired population.

NOTE: Countries are ordered by 2003 average score. The test for significance between the United States and the international average was adjusted to account for the U.S. contribution to the international average. The tests for significance take into account the standard error for the reported difference. Thus, a small difference between the United States and one country may be significant while a large difference between the United States and another country may not be significant. Parentheses indicate countries that did not meet international sampling or other guidelines in 2003. Countries were required to sample students in the upper of the two grades that contained the largest number of 13-year-olds. In the United States and most countries, this corresponds to grade 8.

In addition to teaching students these topics and how to solve specific mathematics problems, the study of mathematics can also build other important skills, such as collaboration and reasoning, strategies to approach and analyze different situations and apply analytical information to them, and a lifelong love of learning. All of these skills are crucial to the success of our students and our society.

How Strong Is the Mathematics Performance of Our Students?

The content outlined above as important for all students is typically covered in Algebra I, Geometry, and Algebra II courses, though data analysis and statistics is sometimes treated as a stand-alone course and at other times integrated within other courses. Whatever the particular course title, not enough high school students are even taking the courses that will expose them to this content. In 2000, only 62% of all high school graduates completed three years of mathematics in ways that effectively exposed them to this content.[8] This means that our students simply are not acquiring the skills and knowledge they need to be successful in the world beyond high school. Indeed, of the more than 70 percent of students who go to college immediately after high school, at least 28 percent must take remedial courses in English or mathematics; more than 60 percent of employers rate the grammar, spelling, writing and basic mathematics skills of high school graduates as "fair" or "poor."[9] Twenty-three percent of fourth grade public school students and 32 percent of eighth grade students—fully 1/5 of fourth graders and 1/3 of eighth graders—scored below basic in mathematics on the 2003 National Assessment of Educational Progress (NAEP).[10]

It is important to note that these scores on NAEP, just as scores on TIMSS, represent an improvement from the past. All 43 states and jurisdictions that took part in the 2000 and 2003 administrations of the 4th grade mathematics NAEP saw increases in average scores. Of the 42 states and jurisdictions that took part in both the 2000 and the 2003 NAEP administrations at the 8th grade level, the average scores of 8th graders rose in 28 states and in the nation, and no state scores declined from 2000 to 2003.[11] The average scores of students at all levels of performance increased; thus, changes seen on average are reflected in changes in the performance of lower-, middle-, and higher-performing students.[12] These improvements are encouraging, but the scores are still disappointing indications as to the success of our combined efforts in raising all children's performance to high standards. We simply are not raising student achievement enough to keep pace with the changing economy and our country's civic needs.

Of great concern, too, is the fact that there has been little to no progress in closing gaps between students in the top and bottom quartiles, between majority and minority students, and between students eligible and ineligible for free or reduced-price lunch. In the 8th grade, the gap between white and black students was no different in 2003 than it was in 1990. The gap between white and Hispanic 8th graders was not statistically significant from any of the previous assessment years.[13] The gap in scores between students who were eligible and those who were not eligible for free or reduced-price lunch (a standard measure of poverty in schools) in the 8th grade did not change significantly in 2003 compared to any of the previous years. While the gap based on eligibility for free or reduced-price lunch in 4th grade decreased from 2000 to 2003, the gap in 2003 was not significantly different from the gap in 1996.[14] 2003 TIMSS data are more encouraging related to the progress made by students who have traditionally underperformed, but nonetheless continues to point to gaps in achievement between advantaged and disadvantaged students and communities.[15]

In summary, the mathematics achievement levels of our students are not where they need to be when compared to our standards, when looking across sub-groups of students, when looking across the performance of their international peers, or when looking at their ability to perform in post high school settings. National productivity and security demands that we create a workforce and a citizenry broadly able to function in the information age, which means that we must do a better job of investing in educating *all* children to achieve to high levels in mathematics, as well as other core subject areas.

The Conditions Required to Improve Student Performance

In order to redesign mathematics education in a way that will result in the level of student achievement needed for today's world, four conditions must be present. These four conditions are described briefly below and are explored in greater detail in the subsequent papers in this series.

High Quality Standards, Curriculum, Instruction, and Assessment: Curriculum needs to be designed around ambitious standards that reflect the current and future needs of our society; it needs to address not only the content to be learned, but also the kind of instructional approaches that are most likely to help students learn the content well. High quality assessments are also needed to provide the kind of data needed to improve instruction and to track student achievement over time in mathematics. Finally, high-level courses must be available equally to all students, and all students must be equally prepared to take them. The second paper in this series addresses these components of standards-based reform in mathematics.

High Quality Teachers for Every Student in Every Classroom: Mathematics educators do not believe they can do their job well if they do not understand the basic core of mathematics. Teacher quality in general, and in the area of mathematics specifically, has multiple dimensions: talented people need to be recruited into the profession; they need to be well-prepared in both content and pedagogical knowledge and skill; they must be recruited to the schools and districts where they are most needed; once there, they need to be assigned to schools and classrooms in ways that lead to equal opportunities for students and equal levels of mathematics achievement; and they need to receive on-going professional development and support in order to continually improve their ability to teach students well. The third and fourth papers in this series address teacher preparation and professional development and teacher recruitment, retention and assignment.

Effective Educational Technology: Technology should support, but can never replace, good instruction. Students should be prepared to use all of the modern tools at their disposal, but it is crucial that they understand the underlying reasoning and computations being used to solve a given problem. Teachers should have available to them instructional tools based on state-of-the art technologies, as well as tools that help them to manage their work and deepen their skills. Issues related to educational technology will be woven throughout this series of papers.

Supportive Partnerships: Multiple partners will need to work together if we are going to transform mathematics education and the resulting student achievement in this area. These partners include federal, state and local policymakers; state education agencies; districts and schools; higher education; business and industry; and parents and other community organizations and leaders. The fifth paper in this series identifies partners and the unique strengths, opportunities, and challenges that each partner brings to the work of improving mathematical literacy.

Endnotes

1. American Diploma Project. (2004). *Ready or not: Creating a high school diploma that counts.* (Washington, DC: Achieve, Inc.). *Ready or Not* focuses on needed proficiency levels in English, as well as mathematics. For further explanation of the methodology used in this report, see pages 105-110.

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3. Reich, Robert. (2000). *The future of success: Working and living in the new economy*. New York: Random House; Reich, Robert. (July, 2004) *Understanding the high performance economy and a vision of what student performance might be*. Speech presented at the CCSSO Summer Institute, Cambridge, MA.; Reich, Robert. (2003). *Nice work if you can get it. The American Prospect Online Edition.* http://www.prospect.org/web/page.ww?section=root&name=ViewWeb&articleId=1417. Retrieved April 10, 2005.

4. Greenspan, Alan. (2004, March 11). *The changing nature of the economy: The critical roles of education and innovation in creating jobs & opportunity in a knowledge economy*. Testimony before the Committee on Education and the Workforce, U.S. House of Representatives.

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6. Gates, William. (2005, February 26). From a speech made to the National Governors Association/Achieve Summit. http://www.gatesfoundation.org/MediaCenter/Speeches/BillgSpeeches/default.htm. Retrieved March 17, 2005.

7. United States Department of Education, National Center for Education Statistics. (2004). *TIMSS 2003 results*, http://nces.ed.gov/timss/Results03.asp?Quest=3. Retrieved February 5, 2005. NOTE: Some argue that international comparisons are neither fair nor valid, because the United States expects to educate every child, regardless of background or future plans. Yet, data from the 1999 TIMSS suggest that the United States is indeed not doing as good a job educating disadvantaged students as other countries. "If the United States educated its disadvantaged students as well as Sweden, it would score above five wealthy countries that it currently performs below." From Nelson, Deborah I. *What explains differences in international performance? TIMSS researchers continue to look for answers*. (2003, September). CPRE Policy Brief No. RB-37. (Philadelphia: Consortium for Policy Research in Education, University of Pennsylvania). p. 4.

8. Barth, Patte. (Winter 2003). A common core curriculum for the new century, in *Thinking K–16. 7*(1). (Washington, DC: The Education Trust).

9. American Diploma Project. (2004). p. 3.

10. U.S. Department of Education, National Center for Education Statistics. (2003). *The nation's report card: Mathematics highlights 2003.* (NCES 2004-451), p. 1.

11. U.S. Department of Education. (2003). p. 3.

12. U.S. Department of Education. (2003). p. 3.

13. U.S. Department of Education. (2003). p. 13.

14. U.S. Department of Education. (2003). p. 15.

15. U.S. Department of Education. (2004). p. 20.

Other works used as background for this briefing paper include

- Sutton, John, & Krueger, Alice, editors. (2002). *ED thoughts: What we know about mathematics teaching and learning.* (Aurora, CO: Mid-continent Research for Education and Learning).

- Kilpatrick, Jeremey, Swafford, Jane, & Findell, Bradford, editors. (2001). *Adding it up: Helping children learn mathematics.* (Washington, DC: National Academy Press).

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While the analysis and suggestions in this briefing paper are informed by the best collective thinking of that group of teachers, administrators, policy makers, product developers, and researchers, the primary authors of this paper are Circe Stumbo and Susan Follett Lusi. Jamie Poolos is the paper's editor. The authors wish to thank the many reviewers of the original drafts of this paper, including Lisa Brady-Gill and Richard Schaar of Texas Instruments Incorporated Michael DiMaggio and Rolf Blank of the Council of Chief State School Officers, and Patricia I. Wright.

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