Math Objectives

- Students will discuss and use multiple ways to answer financial questions involving loans, compound interest, amortization tables, saving money using annuities, and depreciation of assets.
- Students will then apply this finance knowledge to real world situations.
- Students will try to make a connection with how to understand these topics in IB Mathematics courses and on their final assessments.

Vocabulary

- Amortization
- Compound Interest
- Depreciation
- Annuities

About the Lesson

- This lesson is aligning with the curriculum of IB Mathematics Applications and Interpretations HL and IB Mathematics Approaches and Analysis SL/HL
- This falls under the IB Mathematics Content Topic 1 Numbers and Algebra:
  1.4 (AA and AI) Financial applications of geometric sequences and series: Compound Interest and Annual Depreciation.
  1.7 (AI only) Amortization and annuities using technology.
  As a result, students will:
- Apply this information to real world situations.

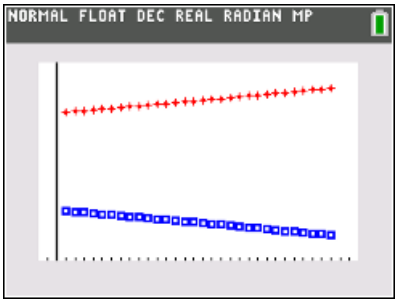
Teacher Preparation and Notes.

- This activity is done with the use of the TI-84 family as an aid to the problems.

Activity Materials

- Compatible TI Technologies: TI-84 Plus*, TI-84 Plus Silver Edition*, TI-84 Plus C Silver Edition, TI-84 Plus CE

** with the latest operating system (2.55MP) featuring MathPrint™ functionality.*



Tech Tips:

- This activity includes screen captures taken from the TI-84 Plus CE. It is also appropriate for use with the rest of the TI-84 Plus family. Slight variations to these directions may be required if using other calculator models.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:
Student Activity
Amortization_Student-84CE.pdf
Amortization_Student-84CE.doc

Before this activity is started, some background knowledge must be given on amortization tables. Amortization tables are quite useful when discussing the payback of loans. They are used as ways of calculating the value for business assets over time. We are going to use them to give you a little taste of what it would cost to purchase a car or a home and how to compare options. Sometime should be spent practicing how to enter values using both the Finance Solver (apps, 1 Finance, 1 Finance Solver) and Amortization Table (Finance Solver, y = , 2nd graph [table]) commands.

Some formulas you will need to be up to date on for this topic are:

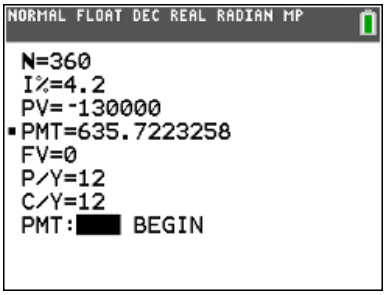
Periodic Payment amount (A): $A = PV \cdot \frac{r \cdot (1+r)^n}{(1+r)^n - 1}$ (rate is divided by 12)

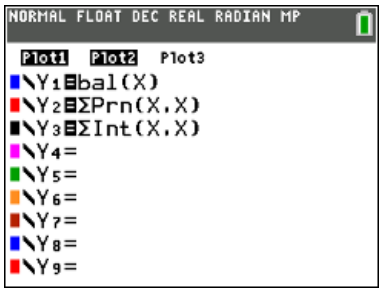
Monthly principal: $Total\ Monthly\ Payment - \left(Outstanding\ Loan\ Balance \cdot \frac{r}{12}\right)$

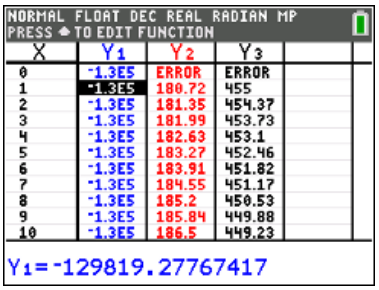
Compound Interest: $FV = PV\left(1 + \frac{r}{100k}\right)^{kn}$

Depreciation: $FV = PV\left(1 + \frac{r}{100}\right)^{n}$

Teacher Note: The teacher should know that this activity will ask students to use their handhelds often. The formulas given above are for reference and can be used as an alternative to the technology used in this activity. During this activity, students should be familiar with finding and using the finance solver and creating an amortization table. You will enter the given loan information into your handheld's Finance Solver and solve for the monthly payment. You will then go to your y = screen and enter bal(X) into Y_1, $\sum Prn$(X,X) into Y_2, and $\sum Int$(X,X). The table will contain the amortization schedule (see example screen shots below).







Problem 1

Daisy and Peter want to purchase a new home and need a loan for $130,000 from the local bank. The loan is for 30 years and the annual interest rate is 4.2%, compounded monthly. They will pay the loan off in fixed monthly installments at the end of each month.

In this problem, we will be answering each part several ways to practice the different capabilities on the TI-84 Plus CE handheld.

(a) Find the amount that Daisy and Peter will be paying back each month by:

(i.) using the Finance Solver on your handheld (show your inputs below).

Solution:

N	**360 or 30 · 12**
I(%)	**4.2**
PV	**-/+ 130000**
Pmt	**+/- 635.72**
FV	**0**
PpY	**12**
CpY	**12**

(ii.) creating an Amortization Table on your handheld through the finance solver, y = screen, and the table resulting from your functions. Fill in the four headings of the table and the first 4 rows of values created on your handheld.

Solution:

Payment Number	Balance after each pmt	Principal Paid per pmt	Interest Paid per pmt
0	**-130000**	**Error**	**Error**
1	**-129819.28**	**180.72**	**455**
2	**-129637.93**	**181.35**	**454.37**
3	**-129455.94**	**181.99**	**453.73**

Explain how you can use the table values to find the monthly payment.

Possible Solution: Students can add the interest paid and the principal paid to find the monthly payment: **455 + 180.72 = 635.72**

454.37 + 181.35 = 635.72

453.73 + 181.99 = 635.72

Teacher Tip: This is a good place to remind students how to input values into the finance solver and amortization table. Knowing when and where to use positive and negative values should be discussed with the students so they understand the meaning behind the signs.

(b) Find the amount that Daisy and Peter will still owe after 10 years by using:

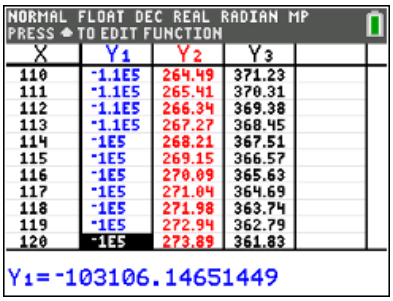
(i.) the Finance Solver.

Solution:

N	**120**
I(%)	**4.2**
PV	**-/+ 130000**
Pmt	**+/- 635.72**
FV	**-103106**
PpY	**12**
CpY	**12**

(ii.) the Amortization Table.

Solution: If you scroll down to X = 120, the value in the Y_1 column gives the answer **103106.**



Teacher Note: There will need to be some discussion here why there may be some inconsistent answers with respect to the two decimal places between the finance solver answer and the amortization table answer. Discuss how the rounded payment used may change the answer.

Reflection

Discuss with a classmate how your TI-84 Plus CE technology has helped through the process of answering **Problem 1**. Also discuss the benefits of both processes that were asked of you in **Problem 1**. Share your results with the class.

Possible Discussion: Finance solver makes finding multiple answers quickly to save students time. The amortization table and interest paid commands show the students the breakdown of the values and gives explanations of the payments.

Problem 2

For problem 2, you will enter the given loan information into your handheld's Finance Solver and solve for the monthly payment. As you practiced in **Problem 1**, you will then go to your y = screen and enter bal(X) into Y_1, $\sum Prn$(X,X) into Y_2, and $\sum Int$(X,X). Go to the table and look at the amortization schedule you have created. You will use this table to aid in the answering of several loan style questions.

Teacher Note: Although the students have done this once or twice already, please take the time to make sure they understand the values that are created in the amortization table. Have one or more students lead the class in entering the correct information along with explanations given.

Christine secures a new 5 year car loan for $21,500 at an annual rate of 6.1%, compounded monthly. She will be making her payments at the end of each month.

(i) Find the total interest paid at the end of the five year loan.

Solution: You can add together the values in the Y_3 column of the table (sum of the interest payments) or you can take the payment you found by using the Finance Solver to attain the amortization table (416.655) and multiply that by the number of payments (60) and subtract the original loan amount. (416.655)(60) = $24999 – 21500 = $3499 in interest

(ii) Find the total amount paid to the bank at the end of the five year loan.

Solution: (monthly payment)(number of payments) = total amount paid
(416.655)(60) = $24999

(iii) If you were to graph the values in Y_2 and Y_3 on the same xy-axes, discuss with a classmate what you would notice about the graph. Discuss your results with the class.

Possible Discussion: As the principal starts in the $300 range it increases slightly each month as part of the monthly payment. The interest payments starts in the $100 range and slightly decreases each month as part of the monthly payment.

(iv) Discuss with a classmate what would happen to the table and graph of data if the amount of the loan increased to $215,000 with the same rate but over 30 years. Share your results with the class.

Possible Discussion: Same as the previous discussion in part (iii). The only exception is that the interest payments per month are drastically higher to start with than the principal payments per month.

Teacher Tip: Teachers may want to take this discussion further and ask the students the meaning behind the interest payments equaling the principal payments.

Further IB Applications

Problem 1:

Emily wants to retire at age 65. She wants to contribute to an annuity fund, which will pay her a monthly allowance of $3000 during her retirement. She wants to save enough money so that the payments last for 20 years. An advisory has told her that she can expect to earn 4.5% interest on her funds, compounded annually.

(a) Calculate the amount Emily would need to have saved in the fund to meet her retirement goal.

Solution:

N	**240**
I(%)	**4.5**
PV	
Pmt	**3000**
FV	**0**
PpY	**12**
CpY	**1**

PV: $477867.35 or $478000

(b) Emily just turned 32. She currently has no retirement savings. Her plan is to save a portion of her salary each month into the fund. Calculate the amount Emily needs to save each month to meet her retirement goal.

Solution:

N	**33 · 12 or 396**
I(%)	**4.5**
PV	**0**
Pmt	
FV	**477867.35**
PpY	**12**
CpY	**1**

Pmt: \$536.36 (\$536.51 if FV of \$478000 is used)

Or you can create an amortization table and add together the values in the Y_2 and Y_3 columns to get \$536.36. (\$536.51 if FV or \$478000 is used)

Problem 2:

In this question, give all answers to two decimal places.

Robert is in the market to purchase a new boat. The price of the boat he is currently looking at is \$15,000, but he cannot afford that amount currently. The boat dealership offers two options to finance a loan.

Finance option A:

A 7 year loan at a nominal annual interest rate of 12% **compounded quarterly**. No deposit required and the repayments are made each quarter.

Finance option B:

A 7 year loan at a nominal annual interest rate of *r* % **compounded monthly**. Terms of the loan require a 10% deposit and monthly repayments of \$225

(a) For option A:

i. Find the repayment made each quarter.

Solution:

N	**7 · 4 or 28**
I(%)	**12**
PV	**-15000**
Pmt	
FV	**0**
PpY	**4**
CpY	**4**

Pmt: \$799.40

ii. Find the total amount paid for the boat.

Solution:

(number of quarterly payments per year) x (number of years) x (payment amount) = total amount paid

(4) x (7) x (799.40) = $22383.20

iii. Find the interest paid on this loan.

Solution: 22383.20 – 15000 = $7383.20

(b) For option B:

i. Find the amount to be borrowed for this option.

Solution: (10%)(15000) = 1500

15000 – 1500 = $13500.00

ii. Find the annual interest rate, *r*.

Solution:

N	**84**
I(%)	
PV	**13500**
Pmt	**-225**
FV	**0**
PpY	**12**
CpY	**12**

I(%): 10.13%

(c) State the option Robert should choose. Justify your answer.

Solution: Option B is the better option if you want to pay less interest, but can afford the 10% down payment. (84 x 225 + 1500 = 20400)

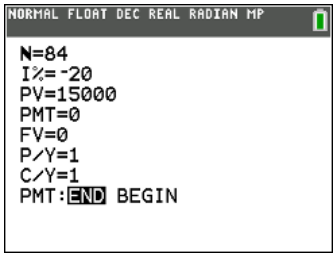
Option A is the better option if you cannot afford a down payment, but are ok with higher monthly payments and paying more interest.

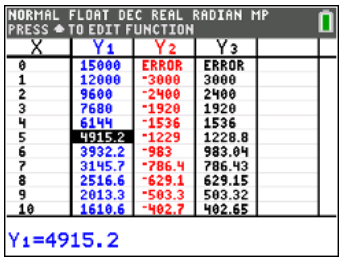
$\left(\frac{799.40}{3} = 266.47\ per\ month,\ \ 266.47\ \times\ 12\ \times 7 = 22383.20\right)$

(d) Robert's boat depreciates at an annual rate of 20% per year. Find the value of Robert's boat 5 years after it is purchased.

Solution: Depreciation = $15000\left(1-\frac{20}{100}\right)^5 = \$4915.20$

Or using an amortization table will give you an answer of $4915.20 as well.





Teacher Tip: Please know that in this activity there is a lot of time dedicated to students talking with one another and sharing their thoughts with the class. The goal here is to not only review Kinematics in calculus, but also to generate discussion.

***Note: This activity has been developed independently by Texas Instruments and aligned with the IB Mathematics curriculum, but is not endorsed by IB™. IB is a registered trademark owned by the International Baccalaureate Organization.*