 Constructing an Ellipse

Name _____

Class _____

Problem 1 – Envelope Construction

In *Cabri Jr.*, open the file *ELLIPSE1*.

- As you drag point *P*, watch the perpendicular line. What do you notice?

Construct the locus of the perpendicular line as *P* travels along the circle.

- What shape does this generate?

Every ellipse has two foci. To explore the other focus, delete or hide the locus you just created.

Reflect point *F* over the *y*-axis. Label the image point *G*. Draw segment *GP*. Then draw a line perpendicular to segment *GP* through point *P*. Drag *P*.

Construct the locus of the new perpendicular as *P* travels along the circle.

- What shape does this generate?

- How do the two loci you constructed compare?

Drag *F* and observe the ellipse.

- Describe the relationship between an ellipse and its foci.

- Describe the relationship between the two foci of an ellipse.

Problem 2 – String and Pins Construction

An **ellipse** is defined as the set of points in a plane such that the sum of the distances from two fixed points (foci) in that plane stays constant.

You can use this definition as another way to construct an ellipse. Open the file *ELLIPSE2*.

Examine the diagram, which contains a segment with a slider and two additional points, *F1* and *F2*, which will become the foci of the ellipse.

The values of *D1* and *D2*, determined by the slider, will be the distances from *F1* and *F2* (respectively) to the point on the ellipse.

Calculate *D3*, the sum of *D1* and *D2*. Drag the slider.

- What is the effect on *D1*, *D2*, and *D3*?

Using the **Compass** tool, construct two circles, one with center *F1* and radius *D1* and another with center *F2* and radius *D2*. Mark the intersections of the two circles. Draw four line segments connecting these points to *F1* and *F2*.

- Hide the circles and drag the slider. What shape do the intersection points make?

Create the loci of the two points as the slider travels along the segment.

- What shape is formed by the loci?

Problem 3 – Deriving the Equation of an Ellipse

Open the picture file *PIC1*. The diagram shows an ellipse with a center at the origin and the beginnings of the derivation. The lengths of the segments connecting (*X, Y*) to the foci are *D1* and *D2*. Let *C* be the distance from the center to the foci. Follow these steps to derive a general equation for such an ellipse.

Clear the diagram and open the picture file *PIC2*. The diagram shows the same ellipse, but the point (*X, Y*) has been moved around the ellipse so that $Y = 0$.

Let *A* be the distance from the center to this point. You should observe the following.

D1 = distance from (*C*, 0) to (*A*, 0) = distance from (−*C*, 0) to (−*A*, 0)
D2 = distance from (−*C*, 0) to (*A*, 0)

1. What is the equation for the distance from (−*A*, 0) to (*A*, 0) in terms of *A*, *D1*, and *D2*?

Go back to *PIC1*.

2. Use the distance formula to write and simplify expressions for *D1* and *D2* in terms of *X, Y,* and *C*.

3. Substitute these into the equation from Step 1.

4. Remove the radicals in the equation by isolating a radical on one side, squaring both sides, and simplifying. Repeat until there are no radicals remaining.

5. Factor out X, then simplify.

6. Divide both sides by $A^2 - C^2$.

Open *PIC3*. Again the diagram shows the same ellipse, but the point (X, Y) has been moved around the ellipse so that $X = 0$. Let B be the distance from the center to this point. This forms two congruent right triangles.

7. What can you conclude about *D1* and *D2*?

8. Use the information from Step 7 to rewrite the equation from Step 4.

9. Use the Pythagorean Theorem to write an expression for B^2 in terms of A and C.

10. Use the expression you just found to rewrite the equation in Step 6.