Leslie Matrices

Student Investigation

7 8 9 10 **11** **12**

TI-Nspire™ Investigation Student 45 min

Introduction

Leslie matrices are a type of transition matrix, designed specifically to model the change in age distribution over time for a population. This activity explores the age distribution of a particular dung beetle. For the purpose of this activity, the discrete one-year age categories are:

- Juvenile (J)
- Adult (A)
- Senior (S)

Juvenile dung beetles do not breed and only have a 40% likelihood of maturing to become an Adult beetle. Adult beetles can breed, producing on average 2 Juvenile beetles per Adult, they have only a 35% probability of maturing to become a Senior beetle. Senior beetles are also capable of breeding, producing on average 1.5 Juveniles per Adult. Senior beetles do not live beyond a year.

Dung Beetle – Warm Up

Dung beetles, as their name suggests, live off the faeces of other animals. As gross as this may sound, in so doing, they perform a very important role, reducing piles of excrement into organic matter, one mouthful at a time. The dung beetle collects a pile of excrement, rolls it up into a ball and then rolls that ball back to its nest, a hole in the ground. The female beetle lays eggs inside this ball. As the eggs hatch the Juvenile beetles eat their way out and eventually grow into Adult beetles, capable of breeding and rolling their own balls of excrement. Needless to say, farmers generally welcome the presence of dung beetles as they clean up and fertilise all at the same time.

Question: 1.

Imagine you're a farmer and want to have these little dung wonders clean up around the stables. Suppose you buy a pile of excrement containing 150 Juvenile beetles only.

a) Use the information provided above to help determine the quantity of Juvenile, Adult and Senior beetles after:

i) One year ii) Two years iii) Three years iv) Four years

b) What do you think will happen to the farmer's population of dung beetles in the long run? Will it thrive, demise or reach some sort of equilibrium?

c) Suppose your ball of excrement came with 150 Juvenile beetles and 100 Adult beetles. Determine the quantity of beetles after one, two, three and four years and comment on your findings.

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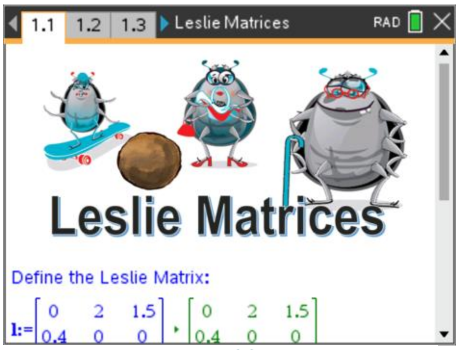
Creating a Leslie Matrix

The previous questions provide some clues as to the repetitive and recursive calculations required to track population growth and distribution.

Open the TI-Nspire™ file: "**Leslie Matrices**".

Page 1.1 consists of a Notes Application. Scroll down to see the Leslie matrix (L) and the population vector or matrix (S).



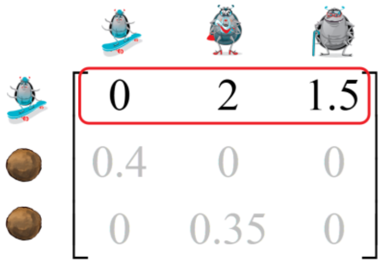
Examine the top row of the Leslie matrix and compare it to the data provided on the previous page.

The **first row**, often referred to as the fecundity row, refers to the reproduction values.

Column 1: Reproduction rate from Juveniles.
Juveniles too young to reproduce new Juveniles

Column 2: Reproduction rate from Adults.
Each adult produces 2 Juveniles

Column 3: Reproduction rate from Seniors.
Each senior produces 1.5 Juveniles



These are the factors that effect the next population of Juveniles.

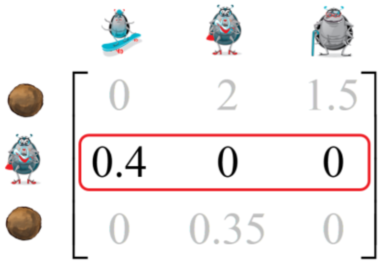
Now examine the **second row** of the Leslie matrix.

The second row represents the transition options into Adult beetles.

Column 1: 40% of Juvenile beetles transition and become Adults

Column 2: 0% of Adult beetles remain as Adults

Column 3: 0% of Senior beetles transition to Adults.



Question: 2.

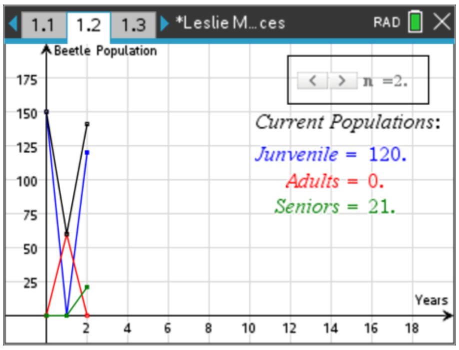
Study the Leslie matrix on Page 1.1 and identify the meaning of the third row.

Navigate to Page 1.2 in the TI-Nspire Leslie Matrix document.

The slider (n) can be used to navigate through time periods where n = 0 represents the initial population for each dung beetle stage.

Use the slider to determine the population of each dung beetle stage from n = 0 through to n = 4.

Check the results against the values you calculated in Question 1.



Question: 3.

Continue clicking on the slider to see what the dung beetle population will be 20 years from the initial population of 150 Juvenile dung beetles. What is happening to the population?

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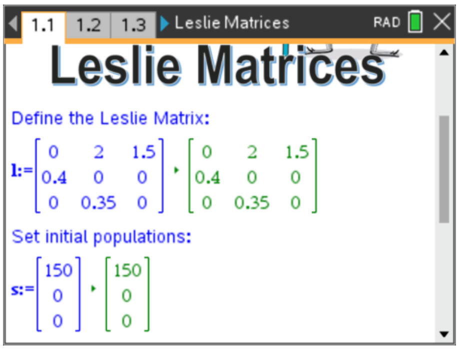
Navigate back to Page 1.1 of the TI-Nspire Leslie Matrix document.

Leslie Matrix and initial population vector are assigned in Maths Boxes. Navigate to the population vector and change the original quantity of Juvenile dung beetles from 150 up to 200.



Question: 4.

Suppose the farmer buys a bigger pile of excrement containing 200 pile of excrement. What happens to this population? Compare the percentage of each age grouping after 20 years and discuss your observations.

Question: 5.

The farmer sources another pile of excrement from a different supplier. This time he has 150 Juvenile beetles and 50 adults. Edit the population vector (matrix S) in Page 1.1 and explore the impact on Page 1.2.

a) Determine the proportions for each age type after 20 years.

b) Comment on the overall population after 20 years.

Question: 6.

This time the farmer places a special order: 130 Juveniles, 50 Adults and 20 Seniors. Edit the population vector on Page 1.1 and explore this result.

a) How does the Population v Time graph compare to previous graphs? Explain

b) Compare the overall population after 20 years with an alternative order of 200 Juveniles.

Question: 7.

A new biologically sound spray has been created to help keep the juvenile dung beetles safe from predators. The spray boosts the survival rate up slightly from 40% up to 50%. [Set the order to 200 Juveniles only.]

a) Determine the proportions for each age type after 20 years.

b) Comment on the overall population after 20 years and the potential long-term dung beetle population.

Extension

The biological balance can be quite delicate. Relatively small changes in the ecosystem can quickly and easily blow out to massive changes, so much so they are sometimes referred to as explosions. Altering the dung beetle age distribution or transition proportions support this notion. Furthermore, taking a small snap shot in time is not always indicative of long-term outcomes. Consider for example the fluctuations in the dung beetle population in the early stages of colonisation, particularly where the age group proportions are not in line with long term predictions.

There are mathematical methods that help predict long-term outcomes, Eigen Values and Eigen Vectors, both can be obtained from the calculator.

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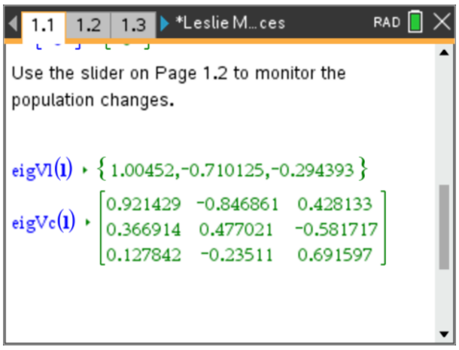

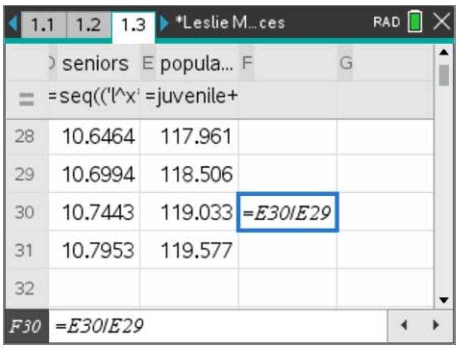

Navigate back to Page 1.1 of the TI-Nspire Leslie Matrix document.

Return the initial conditions to the Leslie Matrix and population vector.

Scroll down to the Maths boxes for:

- Eigen Values (eigVl)
- Eigen Vector (eigVc)

The largest Eigenvalue for the **original** transition information is 1.0045.

The corresponding Eigenvector is located in the corresponding column in the matrix: 0.9214, 0.3669 and 0.1278.

Navigate to Page 1.2 and set n = 30.

Now navigate to page 1.3. This page is a spreadsheet application containing all the raw data for the dung beetles.

Press Ctrl + G and enter the cell reference: F30.

Column E contains the total dung beetle population. Notice that it is increasing slightly each year. To see this grow rate, enter the formula:

= E30/E29

Find the growth rate across E31 and E30.

The Spreadsheet Application on page 1.3 contains a lot of calculations. To force all cells to be recalculated press: **Ctrl + R** (re-calculate). This may be required from time to time to ensure all calculations reflect the most up to date status of each variable.

Question: 8.

Compare the calculated growth rates for the 29th and 30th years with the largest Eigenvalue on Page 1.1, comment on your findings.

Question: 9.

Navigate to cell H1. This group of cells contain the proportion of Juvenile, Adult and Senior beetles in the nth year. Compare these calculated values to the Eigenvector on Page 1.1

Example: From the Eigenvector, Juvenile proportion = $\dfrac{0.9214}{0.0214+0.3669+0.1278} \approx ?$

Question: 10.

Change the transition rate from Juvenile to Adult beetles to 0.35 on Page 1.1

a) What do you notice about the largest Eigenvalue?

b) What do you notice about the overall population on Page 1.2?

Question: 11.

Change the transition rate from Juvenile to Adult beetles to 0.45 on Page 1.1

a) What do you notice about the largest Eigenvalue?

b) What do you notice about the overall population on Page 1.2?

c) Comment on your findings from Questions 9, 10 and 11 with regards to the largest Eigenvalues and Eigenvectors.

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