



Math Objectives

- Students will recognize that samples from a population typically have smaller variability than the population.
- Students will recognize that many reported patterns are really nothing but random noise and not patterns at all.
- Students will recognize that small samples might have more variability than large samples.
- Students will look for and make use of structure (CCSS Mathematical Practices).
- Students will use appropriate tools strategically (CCSS Mathematical Practices).

Vocabulary

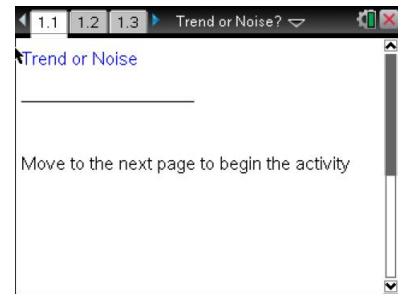
- distribution
- boxplots
- interquartile range (IQR)
- mean
- median
- sample

About the Lesson

- This lesson involves investigating aspects of statistical information reported in the media or other venues, aspects that are often misunderstood by those unfamiliar with sampling.
- As a result, students will:
 - Look at a randomly-generated distribution of student achievement scores for a whole grade and for the individual classes in that grade
 - Analyze boxplots of the data and observe the variation among the classes, identifying a class that seems to have higher scores.
 - Analyze the distribution of the mean scores for each class in the grade and relate it to the distribution of the scores for the entire grade.
 - Choose one of the teachers' classes and examine the scores for that teacher across consecutive years, looking for trends or seemingly large changes.

TI-Nspire™ Navigator™ System

- Transfer a File.
- Use Screen Capture to compare different classes being examined by students.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Tech Tips:

- Make sure the font size on your TI-Nspire handhelds is set to Medium.
- You can hide the function entry line by pressing **ctrl** **G**.

Lesson Files:

Student Activity
 Trend_or_Noise_Student.pdf
 Trend_or_Noise_Student.doc
TI-Nspire document
 Trend_or_Noise.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.



- Use Teacher Edition computer software to review student documents.
- Use Quick Poll to assess students' understanding.

Discussion Points and Possible Answers

Tech Tip: Page 1.2 gives instructions on how to seed the random number generator on the handheld. Page 1.3 is a *Calculator* page for the seeding process. Ensuring that students carry out this step will prevent students from generating identical data. (Syntax: RandSeed #, where # is a number unique to each student.)

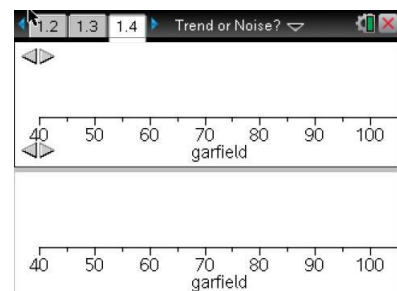
Tech Tip: Once students have seeded their random number generators, they do not have to do it again unless they have cleared all of the memory. But it is important that this be done if the memory has been cleared or with a new device, as otherwise the "random" numbers will all be the same as those on other similarly cleared devices

Teacher Tip: Because students have seeded the handheld with their own number, they will each have a different set of values for the scores for Garfield. This also suggests that the sample answers for the questions will vary considerably depending on the data each student is observing. If students return to Page 1.4 and click the upper arrow, they will generate a whole new set of scores for Garfield.

Teacher Tip: The focus of this activity is on helping students recognize that statements implying a 7% change in the mean or median score signals a change in learning can be misleading. The changes might be due to small sample sizes or just to random chance - or "noise" (a three-year continuous decrease, for example).

Move to page 1.4.

1. a. Use the top arrow in the top work area to display a randomly generated set of mathematics achievement scores for 120 5th grade students at Garfield Elementary School for a given year. Describe the distribution in the dotplot.





Note: Moving the cursor over a point in a dotplot will display the value of that point; moving the cursor over a boxplot will display the five values used to construct the plot.

Sample Answers: The scores range from 50 to 100 with most of them from about 74 to 92. The center of the distribution seems to be between 75 and 85.

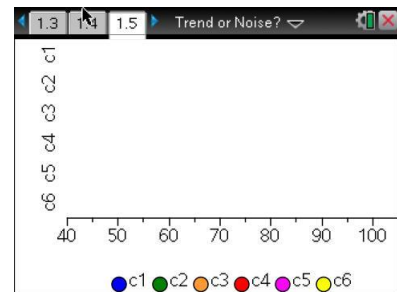
b. The bottom work area shows the boxplot of the scores. Interpret the interquartile range and what it indicates about the scores for the fifth graders at Garfield.

Sample Answers: The IQR is 24, from 65 to 89, which means about half of the students scored between 65 to 89 on the test.

Move to page 1.5.

The scores are also reported by individual classes. Each class has 20 students, each taught by a different teacher.

2. Examine the boxplots of the scores of the six classes.
 - a. Do any of the classes seem to have higher achievement scores than the others? Explain your reasoning.



Sample Answers: Class 5 seems different from the others; it has the highest median, 83, and all of the students in the class scored better than half of the students in three of the other classes (all but class 3). It also has the smallest amount of variability with an IQR of 6.5.

b. Compare the distributions of the classes.

Sample Answers: The distributions for classes 1 and 5 are fairly symmetric. Class 2 is slightly skewed right, and class 6 is slightly skewed left. Class 4 is skewed right. The class scores seem pretty different. The medians of the classes other than class 5 are from 70 to 84.5, so in one class half of the students scored below 70. At least one student in class 1 was the highest scoring student in all of the classes, but class 1 also had the largest IQR, 15.5. Two students in class 6 were outliers and scored the lowest of all the students; one student in class 1 was an outlier. Overall, it looks like class 4 is the second highest scoring class after class 5; three fourths of the students in class 4 scored 78 or higher.



- c. Use the shape of the distribution and the median to estimate the mean scores for each of the classes. Fill out the table for each class.

Sample Answers:

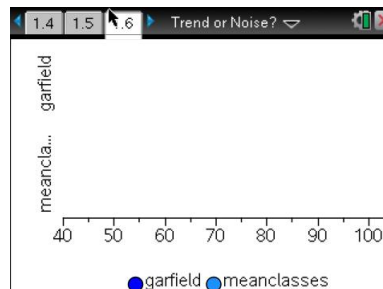
Class	median	Estimated mean	Reasoning about the distribution to support estimate
C1	70	70	symmetric so assume the mean and median are close
C2	77.5	81	skewed slightly right so assume mean larger than median
C3	77.5	74	slightly skewed left so assume mean smaller than median
C4	75.5	80	skewed to the right so assume mean larger than median
C5	84.5	85	symmetric, assume median and mean about the same
C6	85	82	Skewed slightly to the left assume mean smaller than median

- d. Suppose the school administration had a boxplot of the mean scores for each class. Would you expect to see the whiskers of the boxplot as low as 45, the low score for class1? Why or why not?

Sample Answers: A boxplot of the means will not show either the low or the high scores for an individual class as these extremes will be averaged away when the scores are summarized by the mean.

Move to page 1.6.

3. The graph in the top work area displays the scores of all the Garfield fifth graders. The graph in the bottom work area displays the set of the mean scores of each of the classes.
- Remember that class 5 seemed to be different from the other classes. Make a conjecture about the mean of class 5. Explain your reasoning.



Sample Answers: Class 5 probably has the highest mean as it had the almost the highest median and the distribution is relatively symmetric, so it is probably the maximum point in the boxplot. It might be the class with students who are supposed to be "more advanced."



Teacher Tip: Boxplots are not typically used to represent a small set of data. To keep this manageable for students, the boxplots on Page 1.6 are based on only six data values, but the comments in the activity about the interpretation are valid for any boxplot of means.

- b. Make a sketch of the two plots based on the .tns file, indicating the five summary numbers for each.

Sample Answers: Sketches will vary but will typically have similar medians and IQRs around 15 and 7.

- c. Compare the IQR for the two plots, and explain why they are so different.

Sample Answers: The medians of the boxplots are about the same, around 79, but the IQRs are quite different. The IQR for Garfield is 17, while the IQR for the set of means is 7.4. The IQR for the means only displays the mean or average of each set of class scores and not all of the scores. Thus, the values used for the boxplot in the bottom work area are summary values based on the original data and not the data themselves.

- d. Why are the top and bottom scores in the boxplot of the school scores not visible in the boxplot of the means for each class?

Sample Answers: The mean is the average of the scores in a class; thus the top and bottom scores almost "cancel" each other out to produce a value somewhere in the middle. So none of those scores are visible as independent scores in a plot of the means.

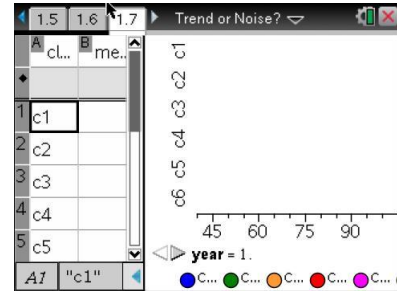
Teacher Tip: Be sure students understand what the mean does to large and small values. You might remind them that the mean encompasses the notion of "fair share" where those with the most give to those with the least, so that in the end, all of the values are the same. It is not about individual values but about the mean as giving the overall picture or summary of the data values.

TI-Nspire Navigator Opportunity: *Screen Capture*
See Note 1 at the end of this lesson.



Move to page 1.7.

4. Column B of the spreadsheet shows the mean scores for each of the classes, and the graph shows the boxplot of the scores from Page 1.5.



- a. Choose one of the classes, and record the indicated information about that class in the table below, filling out only the first row for now.

Sample Answers:

Class 6						All Fifth Grade Classes	
Year	mean	med	min	lqr	max	med	IQR
1	76.2	79	56	24	97	79.5	16.5
2	77	76.5	57	27	97	79	17
3	74.55	76	55	33	98	81	23.5
4	76.4	78	55	10.5	100	81	18.5
5	78.15	79	55	24	100	82	19.5
6	80.15	83	58	21.5	99	82	21

- b. Suppose you had the scores for Garfield fifth graders for the next year. Make a conjecture about how the graph on Page 1.6 would change using those scores.

Sample Answers: I don't think the plots will be very different.

- c. Click on the arrow to display the scores for the next year for all of the Garfield fifth graders. Return to Page 1.6, which now displays distributions from that year. Look at the sketch you made for question 3b. Did the results support your conjecture in 4b? Why or why not?

Sample Answers: The graphs supported my conjecture, but I was surprised to see the medians of both plots were around 78 instead of 81 as in my sketch.



5. In the second row of the table on the previous page, record the information about the achievement scores for the next year for the teacher's class you choose in 4a.
 - Return to Page 1.6 to find the information about all of the fifth grade scores for that year, and record it in the table.
 - Return to Page 1.7. Click the arrow to generate the scores for at least five more successive years, and complete the table. Remember to go to Page 1.6 to obtain the scores for the entire grade for each year.

Teacher Tip: You might want to do this as a whole class to be sure that students follow the directions for filling out the table. Or you might want to have students work in pairs, where one of them records the values in the table and the other generates the next set of scores.

- a. Use the information in the table to comment on how the scores vary for the class you chose.

Sample Answers: The maximums and minimums are relatively constant, from 55 to 58 and from 97 to 100 respectively. But the mean varies nearly 6 points, the median varies 7 points, and the IQR varies 22.5 points.

- b. Compare the overall fifth grade scores over the years to the individual class scores.

Sample Answers: The grade scores have much less variability; the median goes from 79 to 82 while for the class it goes from 76 to 83. The IQR for the grade varies 7 points while for the class it varied by 22.5 points.

- c. Look carefully at the data in the table, remembering they represent the achievement scores in consecutive years for the fifth graders in one teacher's class at Garfield. Identify any "trends" you observe across the years or relatively large differences between two consecutive years.

Sample Answers: Students might notice an increasing trend over the years; for example, in the sample table, the scores increase consistently from year 3 to year six, showing an improvement of nearly 6 points. Or they might note a large increase or decrease from year to year such as the 4-point increase in the median scores for year 5 to year 6.



- d. What explanation might you give for the trends or changes you observed in the data in your table?

Sample Answers: Students should recognize that because all of the scores were generated randomly, all the changes they observed were due to chance, to random behavior.

TI-Nspire Navigator Opportunity: *Quick Poll*

See Note 2 at the end of this lesson.

Wrap Up

Upon completion of the lesson, the teacher should ensure that students are able to understand:

- Samples from a population have smaller variability than the population.
- Small samples can have a large variability from sample to sample.
- Trends can appear in samples collected over time, particularly small samples, that are due to chance.

Assessment

Comment on the following statements that might be made about achievement scores in the Garfield fifth grade:

1. The teacher in class A must be good because she had higher scores than any other class for three years in a row.
2. The new program made a real difference in the fifth grade as achievement rose two points from last year.
3. The new test was a mistake as the median achievement score in the class piloting the test dropped by five points.
4. The scores in each class seem to vary considerably each year, depending on the students.

Answers: Any statement about a trend observed from small samples over a short period of time has to take into account the fact the variability that can occur by chance. Thus, the first three statements would be misleading without further information.



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Note 1

Question 3d, Name of Feature: Screen Capture

A Screen Capture can be taken at this point to compare the different distributions for the fifth grade as a whole and the distribution of the means of the individual classes.

Note 2

Question 5, Name of Feature: Quick Poll

A Quick Poll can be given at the conclusion of the lesson. You can save the results and show a Class Analysis at the start of the next class to discuss possible misunderstandings students might have.