First Principles

Student Activity

7   8   9   10   **11**   12

TI-30XPlus MathPrint™   Worksheet   Student   45 min

Gradient at a Point

A graph of $y = x^2$ is shown below. The graph on the right represents the highlighted region. We are interested in the gradient of the curve at the point (1, 1). The gradient of the secant from (1,1) to (2, 4) can be used to estimate the gradient at the point (1, 1) and allows for a traditional approach for calculating the gradient.

Gradient = $\dfrac{y_2 - y_1}{x_2 - x_1}$ or using function notation: $\dfrac{f(x+d) - f(x)}{(x+d) - x} = \dfrac{f(x+d) - f(x)}{d}$
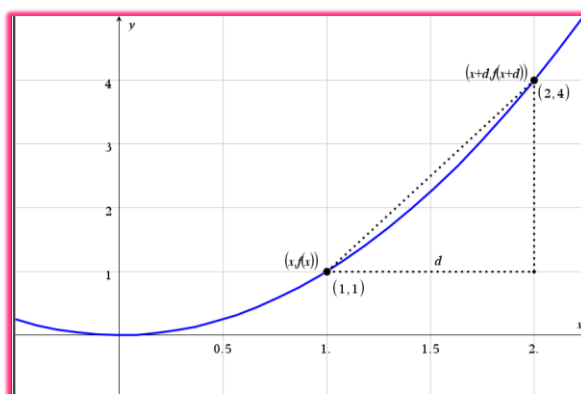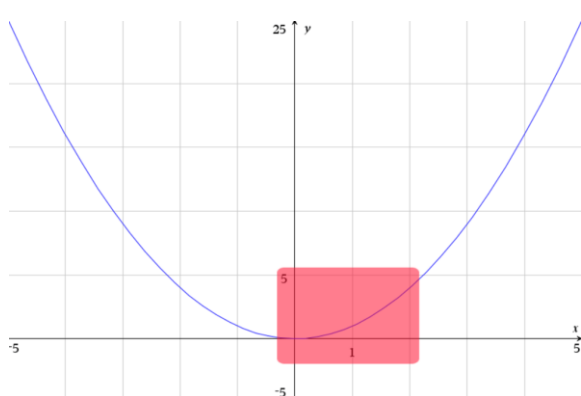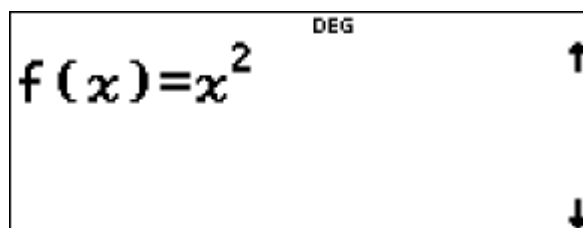

Question: 1.

i) Calculate the gradient of the line joining the points (1, 1) and (2, 4).

ii) Will your answer to part (i) be more or less than the gradient at the point (1, 1)?

iii) The point (2, 4) is moved to (1.5, 2.25). Will the gradient of the secant provide a better (or worse) approximation for the gradient at the point (1, 1)?

iv) Determine the gradient of the secant joining the points (1, 1) and (1.5, 2.25).
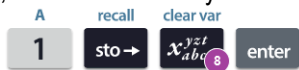
Using your calculator – Gradient at a point

Define the function $f(x) = x^2$ on your calculator.
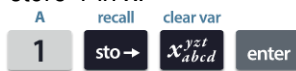
expr-eval   A   clear var   ✓
[table] [1] [x_{abcd}^{yzt}] [x^2] [enter]

Exit the function editor and return to the main screen.

quit
[2nd] [mode]

$$f(x) = x^2 \qquad \text{DEG}$$

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TEXAS INSTRUMENTS

To begin, store '1' in memory location: d.

$$1 \to d$$

And also store 1 in x.

$$1 \to x$$
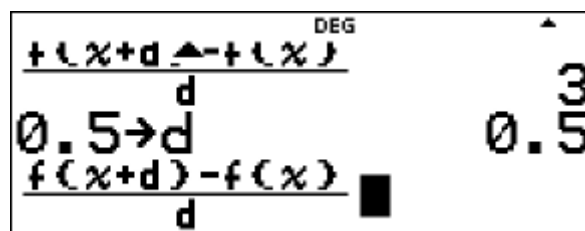
Enter the expression for the gradient using the fraction key and use the corresponding function.
Check your answer to Question 1(i)

$$\frac{f(x+d) - f(x)}{d}$$
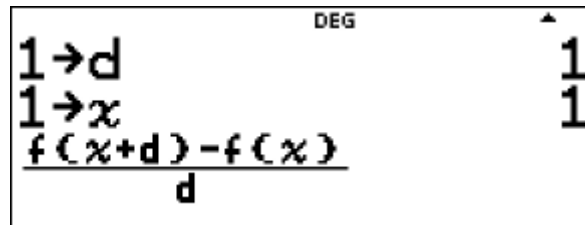
Now store 0.5 in d, then copy and pasted the gradient equation and recalculate.
Compare your result to Question 1(iv).

$$\frac{f(x+d) - f(x)}{d}$$
$$0.5 \to d$$
$$\frac{f(x+d) - f(x)}{d}$$

Question: 2.

Use your calculator to explore the gradient of the secant as *d* gets smaller and smaller and the two points get closer and closer together.

d	1	0.5	0.1	0.01	0.001
Gradient of Secant					

Question: 3.

As *d* gets smaller, what is the gradient of the line passing through (1, 1) approaching?

Question: 4.

Repeat the calculations from Question 2 to explore the gradient on the curve of the point (2, 4).
Change the stored value for x to 2.

d	1	0.5	0.1	0.01	0.001
Gradient of Secant					

Defining a Function for the Gradient

With d set to 0.001, the gradient of the secant provides a good estimate for the gradient of a point on the curve.
Use table key to return to the function editor. Navigate to g(x) and define it the same as the gradient from the calculator's home screen.

$$g(x) = \frac{f(x+d) - f(x)}{d}$$
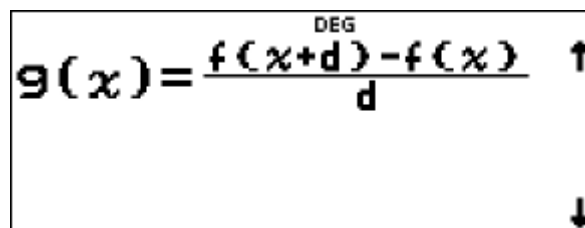
Once the gradient function is defined press: enter

TEXAS INSTRUMENTS

A table of values can be generated automatically for both f(x) and gradient which is now defined in g(x).

Set the table to start at -3 in steps of 1.

DEG
TABLE SETUP   ↑
Start=-3
Step=1
Auto   x = ?
                          CALC

A sample section of the table is shown opposite. Check to make sure your results are the same. (d = 0.001)

DEG
x	f(x)	g(x)
-3	9	-5.999
-2	4	-3.999
-1	1	-1.999

x=-3

Question: 5.

Scroll through the table of values and study the relationship between x and the gradient of the function $f(x) = x^2$.

i) Graph the function $f(x) = x^2$ on the axis provided.

ii) Plot the points corresponding to the gradient of the function.

iii) Write a rule for the gradient function in the form: $f'(x) =$ where $f'(x)$ represents the gradient of the function $f(x)$.
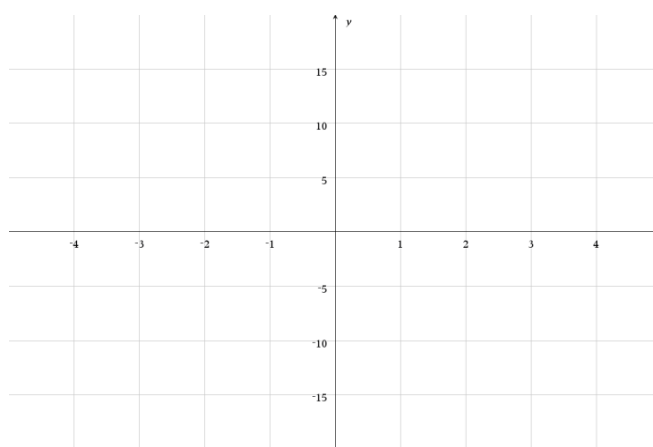
Question: 6.

Change the defined function on the calculator to: $f(x) = x^2 + 1$. Explore the table of values for the gradient function and compare it to the gradient function for $f(x) = x^2$. Suggest reasons for the result.

Question: 7.

Change the defined function on the calculator to: $f(x) = (x-1)^2$. Explore the table of values for the gradient function and compare it to the gradient function for $f(x) = x^2$. Suggest reasons for the result.

Question: 8.

Change the defined function on the calculator to: $f(x) = x^3$. Explore the table of values for the gradient function, graph the results and determine a function for the gradient.

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