

## About the Lesson

In this activity, students will run two experiments that involve simulating pouring out coins from a bag. They will collect and graph data, and they will use different methods to find equations that model the data. Then students will find the inverse of the data and the corresponding equation to model it. As a result, students will:

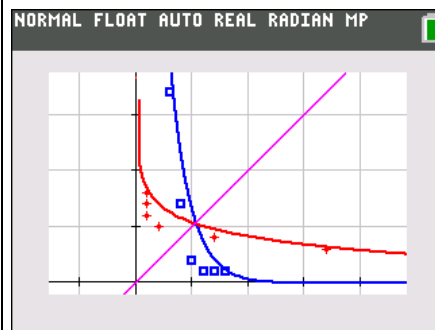
- Model and identify the properties of exponential decay.
- Express the inverse of a given exponential function as a logarithmic function and graph it.
- Show that the graphs of the exponential and logarithmic functions are reflected images in the line  $y = x$ .
- Analyze a graph of data to determine a function that relates two quantities in a real-world scenario.

## Vocabulary

- exponential decay
- exponential growth
- function
- inverse
- logarithm

## Teacher Preparation and Notes

- It would be beneficial for students to clear all lists and functions. Press **[2nd]** **[MEM]** and select **ClrAllLists**. Press **[Y=]**, move to any equation that is defined and press **[CLEAR]** on each equation.
- Prior to beginning the activity, students should have some experience with the general form of an exponential function  $f(x) = a \cdot b^x$ , understand the properties of logarithms, and also be familiar with properties of inverse functions.



### Tech Tips:

- This activity includes screen captures taken from the TI-84 Plus C Silver Edition. It is also appropriate for use with the TI-84 Plus family with the latest TI-84 Plus operating system (2.55MP) featuring MathPrint™ functionality. Slight variations to these directions given within may be required if using other calculator models.
- Access free tutorials at <http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials>
- Any required calculator files can be distributed to students via handheld-to-handheld transfer.

### Compatible Devices:

- TI-84 Plus Family
- TI-84 Plus C Silver Edition

### Associated Materials:

- CoinToss\_Student.pdf
- CoinToss\_Student.doc
- COINSIM1.8xp
- COINSIM2.8xp

**Tech Tip:** Before beginning the activity, the programs **COINSIM1** and **COINSIM2** need to be transferred to the students' calculators via handheld-to-handheld transfer or transferred from the computer to the calculator via TI-Connect.

Exponential growth and decay are usually encountered with population and half-life problems. This activity uses two programs that simulate tossing coins out of a bag. The data is gathered and analyzed to develop exponential models and then to find the inverse logarithmic functions.

### Problem 1 – Coin Toss Simulation: Exponential Decay

Students first simulate pouring 8 coins out of the bag using the **randInt(** command. A 0 or a 1 will be randomly chosen 8 times. 0 = heads, 1 = tails.

1. How many coins landed heads up? How many coins go back in the bag?

**Answer:** Answers should vary but be around 50% 0s and 50% 1s.

Repeat the simulation, replacing 8 with the number of coins that went back in the bag. Continue to repeat the simulation.

2. How many times is the experiment run until zero coins land tails up (no coins in the bag)? Compare this answer with your partner or other students in the class.

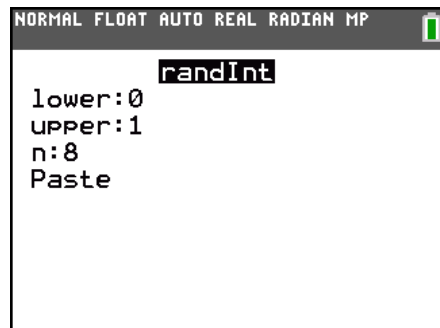
**Answer:** Answers will vary. Discuss why there is difference.

Students are directed to run the program **COINSIM1** and enter in a number between 100 and 999 when prompted. The program simulates pouring out the coins, removing those that land heads up, replacing the rest, and repeating the experiment.


The program will generate three lists of data. The trials are numbered in **L1**. The number of coins poured out of the bag in each trial is given in **L2**. The number of coins removed in each trial is given in **L3**.

3. Press **[STAT]** **[ENTER]** to view the lists. How can you check that the simulation was executed correctly?

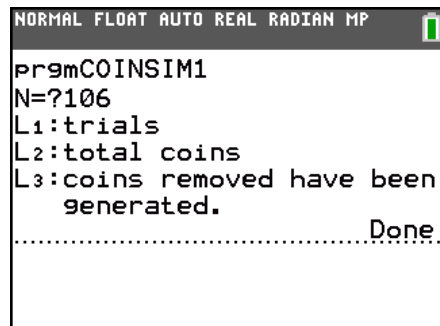
**Answer:** For example, students will see the number they chose at the top of **L2**.



```
NORMAL FLOAT AUTO REAL RADIAN MP
randInt
lower:0
upper:1
n:8
Paste
```



```
NORMAL FLOAT AUTO REAL RADIAN MP
EXEC EDIT NEW
1:COINSIM1
2:COINSIM2
```



```
NORMAL FLOAT AUTO REAL RADIAN MP
prgmCOINSIM1
N=?106
L1:trials
L2:total coins
L3:coins removed have been
generated.
..... Done
```

They should make sense of the other columns.

Students will view the data as a scatter plot. Then, they will find an equation in the form  $y = a \cdot b^x$  to model this data where  $a$  is the initial value and  $b$  is the multiplier. Students should store these values on the HOME screen.

4. What is the value of  $a$ ? Store this value as **A** by pressing **STO** **ALPHA** **A**.

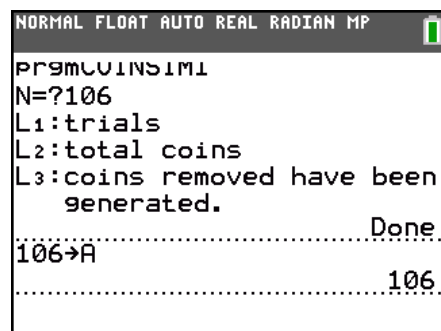
**Answer:** The variable  $a$  is their starting value.

Next, students are to clear out the values in **L3** and calculate the ratios between the values in **L2**. They should continue these calculations until they have used all of the values in **L2**.

The mean value of **L3** can be found by using the **Mean** command. This can be found by pressing **2nd** **LIST**, scrolling over to the **MATH** menu, and selecting **mean**(. This value should then be stored as **B**.

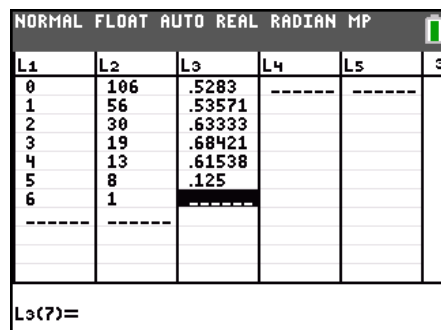
5. What is the value of  $b$ ? Store this value as **B**.

**Answer:** Students should record their value of  $b$  they found from the ratios and stored.



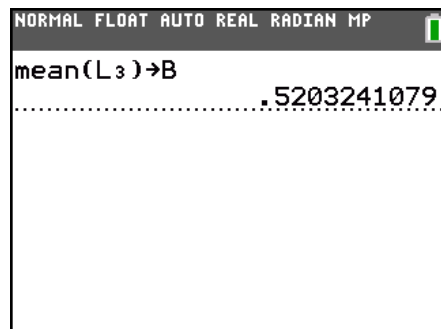
```

NORMAL FLOAT AUTO REAL RADIAN MP
Prgm COINSIM1
N=?106
L1: trials
L2: total coins
L3: coins removed have been
      generated.
..... Done
106 -> A
..... 106
    
```



L1	L2	L3	L4	L5	3
0	106	.5283	-----	-----	
1	56	.53571			
2	30	.63333			
3	19	.68421			
4	13	.61538			
5	8	.125			
6	1				
-----	-----				

L3(7)=



```

NORMAL FLOAT AUTO REAL RADIAN MP
mean(L3) -> B
..... .5203241079
    
```

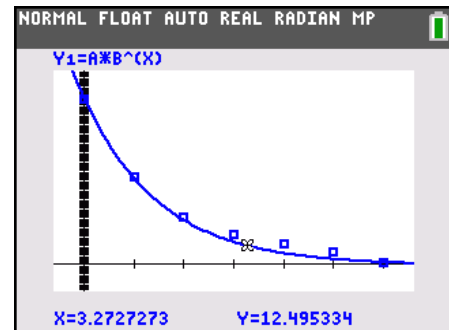
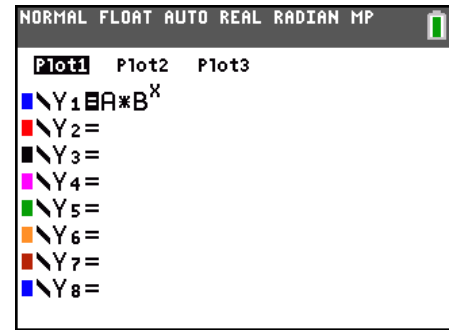
Now, graph your model. Press  $\boxed{Y=}$  and enter  $A*B^X$

6. How well does this function model the data? If it does not fit well, adjust  $b$  until you have a good fit.

Students should assess how well their equation models the data. If it does not fit well, students are to modify their  $b$ -value until they have a good fit.

The students now have a series of questions that are good for a class discussion. Sample responses are in parentheses.

- Is this a growth or decay model? (decay)
- At what percent are the coins being removed? (48% for the given example where the  $b=52$ .)
- Explain why the  $b$ -value is what it is with this experiment. (Since coins have around a 50% chance of coming up heads, that would determine the  $b$ -value.)



Now check your model by performing an exponential regression. Press **[STAT]**, move to the **CALC** menu, and select **ExpReg**.

The command shown performs the regression and stores the resulting equation in **Y2**.

7. How do the values of  $a$  and  $b$  in the exponential regression compare with those of your model?

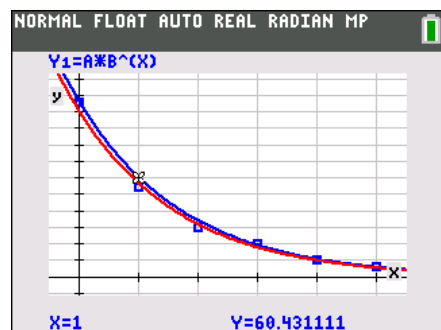
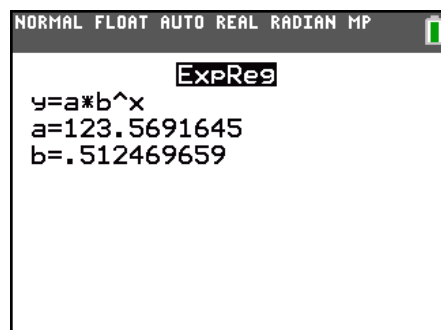
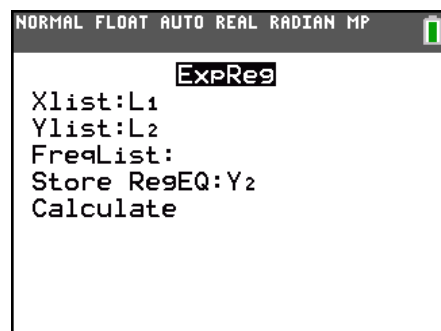
**Answer:** It should agree well. Otherwise students should examine where a mistake was made.

8. How well does the exponential regression fit the data? Describe how you know.

**Answer:** By examining the graph you can see that about the same number of data point lie above the line as those that are below the graph.

Suppose you started with a bag of 100 coins. You want to know how many times you would have to repeat the experiment before less than 50 coins is left. In other words, given a number of coins, how many trials does it take to achieve that number or fewer coins in the bag?

A function that answers this type of question is the **inverse** of the function you just created. Instead of entering the number of trials and getting the number of coins in the bag as an output, you need to enter the number of coins in the bag and get the number of trials as an output.



Now that the students have had two experiences with exponential modeling – modeling themselves and using the exponential regression – the activity turns to exploring the inverse of exponential functions.

As the students have learned before, the inverse reverses the  $x$ - and  $y$ -coordinates of points. Students will view a graph of  $(L2, L1)$ . Their objective is to find the inverse function for this data.

To find the inverse function for an exponential equation, students need to think about exponential functions and their inverses, logarithmic functions.

Exponential functions:  $\text{Base}^{\text{Power}(x)} = \text{Answer}(y)$ .

Logarithmic Functions:  $\log_{\text{Base}} \text{Answer}(y) = \text{Power}(x)$ .

To find the inverse of a function, switch the  $x$  and  $y$  coordinates of the points. This can be done with the scatter plot of this data.

- Delete functions **Y2**. Turn **Plot1** off.
- In the **Plot2** settings, switch the **XList** and **YList**.

Press **[ZOOM]** and select **ZoomStat** to view the plot.

9. What about the model you created? To find its inverse, switch the  $x$  and  $y$  algebraically:

$$y = ab^x \rightarrow x = ab^y$$

To solve the new equation for  $y$ , divide both sides by  $a$ , then take the log of both sides. By applying exponent rules, you should be able to isolate  $y$  on one side.

**Answer:**

To find the inverse function of  $y = a \cdot b^x$ , first switch the  $x$ - and  $y$ -values:  $x = a \cdot b^y$  and solve for  $y$ .

Divide both sides by  $a$ :  $\frac{x}{a} = b^y$

Apply logs to both sides:  $\log \frac{x}{a} = y \log b$

Divide both sides by  $\log b$ :  $y = \frac{\log\left(\frac{x}{a}\right)}{\log b}$

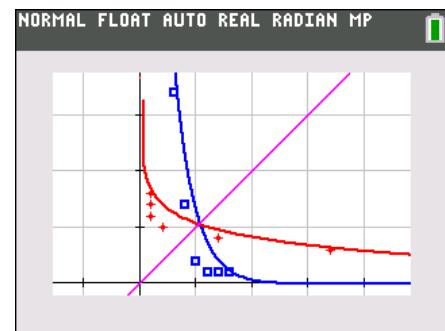
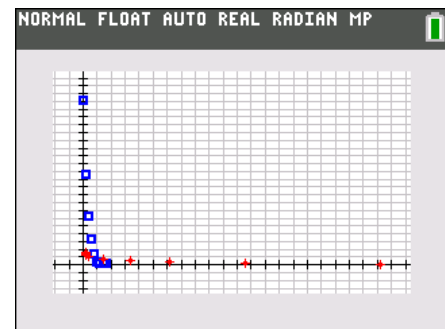
Next, students should type in the equation that was just derived in **Y2**.

10. How well does the log function model the data?

**Answer:** This function is a model of the experiment just as much as the exponential model. It is the same data, viewed “from another perspective.”

What does it mean *graphically* for two functions to be inverses?

- Graph two scatter plots together. Have both Plot1 & Plot2 on.
  - **Plot1:** use **L1** as the **XList** and **L2** as the **YList**
  - **Plot2:** use **L2** as the **XList** and **L1** as the **YList**.  
Choose a different **Mark** to distinguish it from **Plot1**.
- Select **ZoomStat** to view the graph. Because of the scale it may be difficult to see the relationship. Use **ZoomBox** to create a region to zoom into near the origin.



11. What do you notice about the two sets of data?

**Answer:** The two sets of data are mirror images of one another.

12. Imagine if you drew a vertical line through each point in **Plot1** and a horizontal line through each point in **Plot2**. (It may help to have the GridLine turned on. Do this with  $\boxed{2nd} \boxed{ZOOM}$  Format.)

a. Where do you think these lines would cross?

**Answer:** They would cross at the point (1,1).

b. What line could you draw between the two data sets so that one data set was the reflection of the other over that line?

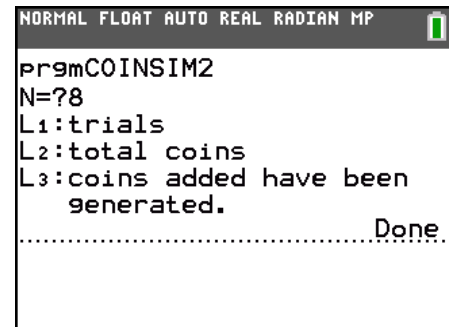
**Answer:** The reflection line is  $y = x$ . They are to graph the reflection line with the data.

### Problem 2 – Coin Toss Simulation: Exponential Growth

The second part of this activity follows the same format as Problem 1 with the exception that students start with four coins and do the reverse process of adding the number of coins that come up heads.

Students will run the program **COINSIM2** and enter in a number between 1 and 10 when prompted.

Again, data will be generated in **L1**, **L2**, and **L3**.



```

NORMAL FLOAT AUTO REAL RADIAN MP
prgmCOINSIM2
N=?8
L1: trials
L2: total coins
L3: coins added have been
    generated.
..... Done.
    
```

Students will repeat the processes from Problem 1 to model the data and the inverse data.

**Note:** This  $b$  value is different than the previous experiment. This is a good opportunity to discuss why.