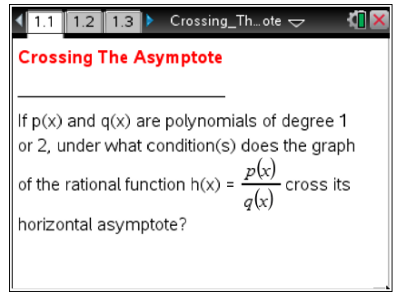


Math Objectives

- Students will test whether the graph of a given rational function crosses its horizontal asymptote.
- Students will examine the relationship among the coefficients of the polynomials in the numerator and denominator of various rational functions whose graph does or does not cross its asymptote.
- Students will use appropriate tools strategically (CCSS Mathematics Practice**).**
- Students will look for and make use of structure (CCSS Mathematics Practice).

Vocabulary

- rational function
- horizontal asymptote
- vertex

About the Lesson

- This lesson involves the graph of a rational function of the form $r(x)=\dfrac{p(x)}{q(x)}$.
- Note: Some portions of the activity require CAS functionality – TI-Nspire CAS Required.
- As a result, students will:
- Discover conditions under which the graph of $y=r(x)$ does or does not cross its horizontal asymptote. The functions $p(x)$ and $q(x)$ are assumed to be linear or quadratic polynomials.
- Manipulate graphs of rational functions and their asymptotes to determine whether they intersect.
- Make conjectures about the relationship between the coefficients of the polynomials in the numerator and denominator of a rational function whose graph does or does not cross its horizontal asymptote.

TI-Nspire™ Navigator™ System

- Send out the *Crossing_The_Asymptote.tns* file.
- Monitor student progress using Class Capture.
- Use Live Presenter to spotlight student answers.

Activity Materials

- Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software

Tech Tips:

- This activity includes screen captures taken from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Materials:

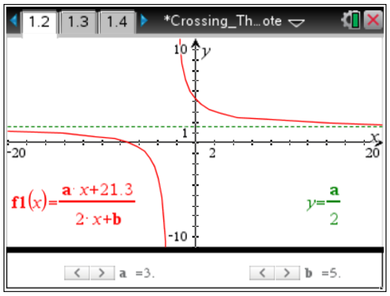
Student Activity

- Crossing_The_Asymptote_Student.pdf
- Crossing_The_Asymptote_Student.doc

TI-Nspire document

- Crossing_The_Asymptote.tns

Discussion Points and Possible Answers



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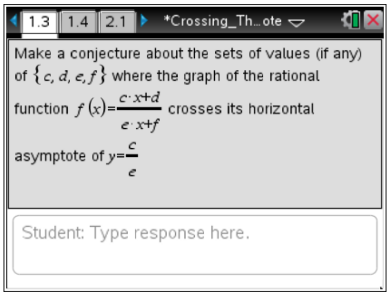
1. Consider an example where when both $p(x)$ and $q(x)$ are linear:
   $p(x) = a \cdot x + 21.3, \quad q(x) = 2 \cdot x + b$ where $a \neq 0$.

Set the value of the slider **b** to -6. Then scroll through the values of slider **a** from -6 to 6, and make a note of the cases when the graph of $y = f1(x)$ crosses its asymptote $y = f2(x)$. Ignore $a = 0$ since we are only considering values of $a \neq 0$. Set **b** to -5, and scroll through the values of **a** noting any cases where the graph crosses its asymptote. Repeat this process for values of **b** from -6 to 6. What pairs of values (if any) for **a** and **b** did you note?

Answer: There are no pairs of values (*a*,*b*). The graph of such a rational function never crosses its horizontal asymptote.

> **Tech Tip:** If a student is unsure whether a graph of a given rational function crosses its horizontal asymptote, suggest that they zoom-in on the graph k times, check whether the graphs intersect, and then press "undo" k times to return to the original graph.
>
> **Tech Tip:** Zoom out on the iPad app by pinching. Do the reverse to zoom in.



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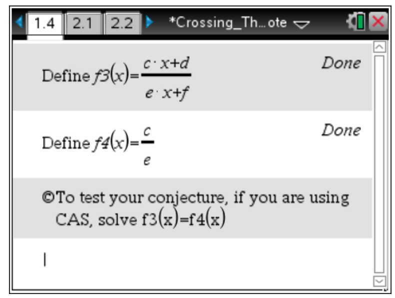
2. In general, make a conjecture about the sets of values (if any) of $\{c, d, e, f\}$ where the graph of the rational function $f3(x) = \dfrac{c * x + d}{e * x + f}$ crosses its horizontal asymptote $f4(x) = \dfrac{c}{e}$. [Assume $c \neq 0, e \neq 0$, and $e \cdot x + f$ is not a multiple of $c \cdot x + d$.] Type your conjecture in the indicated space of Page 1.3.

   Sample Answers: The graph of such a rational function never crosses its horizontal asymptote.

Teacher Tip: Ask students about the reasoning they used to reach their conjecture.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 1 at the end of this lesson.

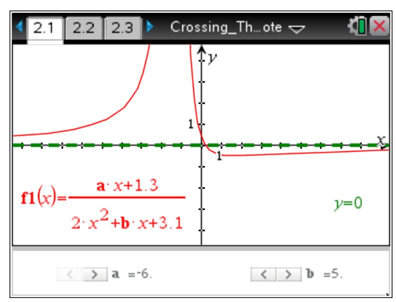
Move to page 1.4.



3. Test your conjecture. The functions $f3(x)$ and $f4(x)$ have been defined. Enter $solve\left(f3(x)=f4(x),x\right)$. Does this result validate your conjecture? Why or why not? If your conjecture was not correct, revise it so the resulting statement is correct.

<u>Answer:</u> "$solve\left(f3(x)=f4(x),x\right)$" gives the response "$false$" meaning this equation never has a solution so the graph of the rational function of this type never crosses its horizontal asymptote. Students can verify this claim algebraically by showing the equation $\dfrac{c*x+d}{e*x+f}=\dfrac{c}{e}$ is a contradiction since $de \neq fc$ [$e\cdot x+f$ is not a multiple of $c\cdot x+d$ is an assumption].

Move to page 2.1.



4. Consider an example where when p(x) is linear and q(x)is quadratic:
$p(x)=a\cdot x+1.3$; $q(x)=2\cdot x^2+b\cdot x+3.1$ where $a\neq 0$.

Set the value of the slider **b** to -6. Then scroll through the values of slider **a** from -6 to 6, and make a note of the cases when the graph of $y=f1(x)$ crosses its asymptote, $y=f2(x)$. Ignore **a** = 0 since we are only considering values of $a\neq 0$. Then set **b** to -5, and scroll through the values of noting any cases where the graph crosses its asymptote. Repeat this process for values of b from -6 to 6. Describe the pairs of values of **a** and **b** that you noted.

<u>Answer:</u> All pairs of values (a,b) work. The graph of such a rational function always crosses its horizontal asymptote, y = 0, at the point whose *x*-coordinate is the root of the linear polynomial in the numerator of the rational function.

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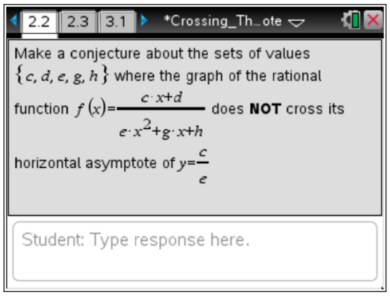

5. In general, make a conjecture about the sets of values $\{c, d, e, g, h\}$ where the graph of the rational function $f3(x) = \dfrac{c * x + d}{e * x^2 + g * x + h}$ does **not** cross its horizontal asymptote $f4(x) = 0$. [Assume $c \neq 0, e \neq 0$, and $e \cdot x^2 + g \cdot x + h$ is not a multiple of $c \cdot x + d$]

   Type your conjecture in the indicated space on Page 2.2.

   Sample Answers: There are no sets of values. The graph of such a rational function always crosses its horizontal asymptote, $y = 0$, at the point whose x-coordinate is the root of the linear polynomial in the numerator of the rational function.

Teacher Tip: Ask students about the reasoning they used to reach their conjecture.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 2 at the end of this lesson.

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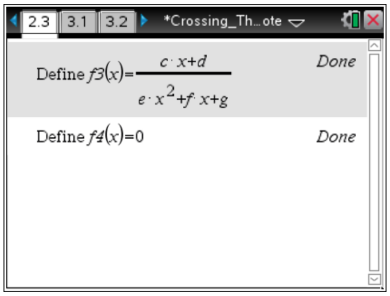

6. Test your conjecture. The functions $f3(x)$ and $f4(x)$ have been defined. Enter $solve\left(f3(x) = f4(x), x\right)$. Does this result validate your conjecture? Why or why not? If your conjecture was not correct, revise it so the resulting statement is correct.

   Answer: "$solve\left(f3(x) = f4(x), x\right)$" gives the response "$x = \dfrac{-d}{c}$" meaning that the graph of such a rational function always crosses its horizontal asymptote, $y = 0$, at the point whose x-coordinate is the root of the linear polynomial in the numerator of the rational function.

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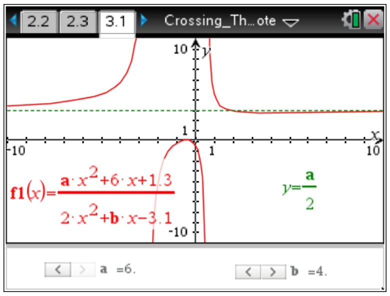

7. Consider an example where when both p(x) and q(x) are quadratic:

$$p(x) = a \cdot x^2 + 6 \cdot x + 1.3\,;\ q(x) = 2 \cdot x^2 + b \cdot x - 3.1$$

where $a \neq 0$.

   a. Set the value of the slider **b** to -6. Then scroll through the values of slider **a** from -6 to 6, and enter the value of ***a*** (if one exists) when the graph of $y = f1(x)$ does **not** cross its asymptote, $y = f2(x)$, in the table. Ignore **a** = 0 since we are only considering values of $a \neq 0$. Then set **b** to -5, and scroll through the values of **a.** Repeat this process for values of **b** from -6 to 6.

Hint: For a given value of ***b***, there is at most one value of ***a*** for which the graph does not cross its asymptote.

Answer:

b	-6	-5	-4	-3	-2	-1	1	2	3	4	5	6
a	-2		-3	-4	-6	*	*	6	4	3		2

   b. The boxes below -1 and 1 are blank. If the values of the sliders for **a** and **b** were not limited, what values would go in each of these two boxes?

Answer: These two entries would be -12 and 12.

   c. Make a conjecture about the relationship between ***a*** and ***b*** that is true for the rational functions in this set whose graph does **not** cross its horizontal asymptote.

Answer: $a \cdot b = 12$

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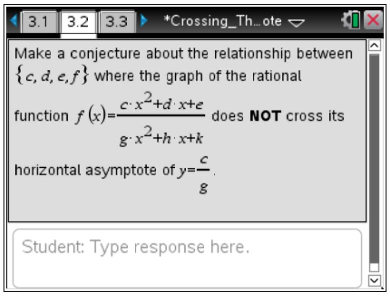

8. In general, make a conjecture about the relationship between $\{c, d, g, h\}$ where the graph of the rational function $f3(x) = \dfrac{c * x^2 + d * x + e}{g * x^2 + h * x + k}$ does **not** cross its horizontal asymptote $f4(x) = \dfrac{c}{g}$. [Assume $c \neq 0, g \neq 0$ and

$g \cdot x^2 + h \cdot x + k$ and $c \cdot x^2 + d \cdot x + e$ do not have a common linear factor.] .

Sample Answers: The graph does **not** cross its horizontal asymptote **when** $c \cdot h = d \cdot g$**.** This condition is a generalization of $a \cdot b = 12$ from Question 7.

Teacher Tip: Ask students about the reasoning they used to reach their conjecture.

TI-Nspire Navigator Opportunity: ***Quick Poll***
See Note 3 at the end of this lesson.

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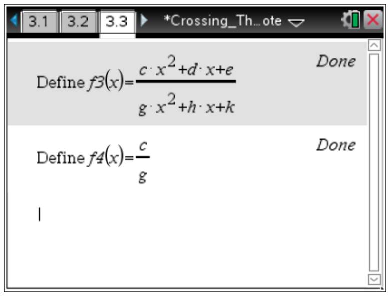

9. Test your conjecture. The functions $f3(x)$and $f4(x)$ have been defined. Enter $solve\left(f3(x)=f4(x),x\right)$. Does this result validate your conjecture? Why or why not? If your conjecture was not correct, revise it so the resulting statement is correct.

Answer: "$solve\left(f3(x)=f4(x),x\right)$"gives "$x = \frac{-(ck-eg)}{ch-dg}$ $or\ g \neq 0$" meaning the equation does not have a solution, or, equivalently, the graph of such a rational function does not cross its horizontal asymptote if $c \cdot h - d \cdot g = 0$ or $c \cdot h = d \cdot g$**.** This condition is the generalization of the condition $a \cdot b$ = 2·6 = 12 from Question 8. The restriction $g \neq 0$ was an assumption.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students are able to understand:

- The graph of a rational function may cross its horizontal asymptote and this phenomenon occurs more often than most people think.

Assessment

- Students could consider the extension of this question and determine which rational functions have graphs that cross their oblique asymptote if $\deg p(x) = \deg q(x) + 1$.
- Students could consider the extension of this question about which rational functions with $\deg p(x) = \deg q(x) = 3$ have graphs that are tangent to their horizontal asymptote.

TI-Nspire Navigator

Note 1

Question 2, Name of Feature: Quick Poll

Send the question on page 1.3 as a Quick Poll to share students' conjectures and generate class discussion about which ones are correct. Do this before students check their answer.

Note 2

Question 5, Name of Feature: Quick Poll

Use a Quick Poll for page 2.2 to share students' conjectures and generate class discussion about which ones are correct.

Note 3

Question 8, Name of Feature: Quick Poll

Use a Quick Poll for page 3.2 to share students' conjectures and generate class discussion about which ones are correct.

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