Buffon's Needle

Student Activity

7   8   **9   10**   11   12

TI-Nspire	Investigation	Student	30 min

Problem

Your pen just rolled off the desk onto the timber floor, as you lean over to pick it up you notice that your pen doesn't cross any of the joins in the floorboards. This brings back memories from your early childhood days of jumping over the cracks in the pavement. It was relatively easy to avoid the cracks in the pavement because your foot was so much smaller than the distance between the cracks. Your pen, on the other hand, seems to be exactly the same length as the distance between joins in the floor boards.

Curious, you pick up your pen and purposely drop it again. This time your pen crosses one of the joins. Not wanting to end on 'bad luck' you drop your pen once again, another hit. This is not good! You try again, this time it lands neatly between the cracks, a miss; good luck has returned. Distracted by these events you sit and ponder; "What are chances that when you drop your pen it will land over a crack?"

Question: 1.

Measure the length of your pen. Draw two parallel lines separated by a distance equal to the length of your pen. Experiment with your pen and then estimate the probability that, when dropped randomly your pen will land over one of the lines.
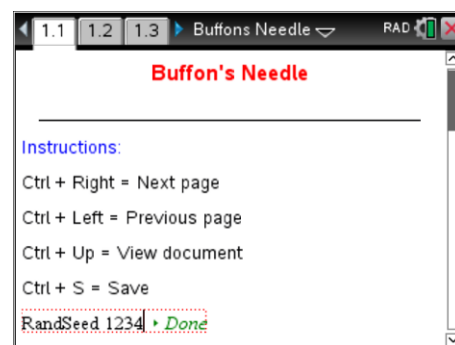Answers will vary.

Teacher Notes:

The purpose of this question is to ensure that students understand that the probability of an event must lie between 0 and 1. The analytical solution to the problem involves calculus, therefore students at this level must estimate their answer; opinions will differ considerably. Once students have estimated their answer they will have a level of 'buy in' towards the result.

Simulating the Event

Open the TI-Nspire file: **Buffon**

Page 1.1 contains a set of instructions. These instructions include setting the random number generator. Change the four digit number to some random four digit number such as the last four digits of your phone number, then press [**ENTER**].

1.1	1.2	1.3 ▶ Buffons Needle ▽ RAD

Buffon's Needle

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Instructions:

Ctrl + Right = Next page

Ctrl + Left = Previous page

Ctrl + Up = View document

Ctrl + S = Save

RandSeed 1234 ▸ *Done*

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Navigate to page 1.2.

Move the mouse over the slider and click to simulate a pen drop. The animation will highlight the line and display a HIT when the pen lands across a line. Try a few to make sure everything is working.

Once you have finished experimenting, change the value of n back to 1, then simulate and record 100 pen drops.
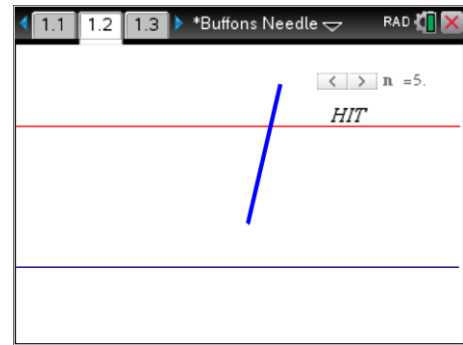
Question: 2.

Record the number of 'hits' out of 100 pen drops.

Answers will vary.

Teacher Notes: Expect answers between 50 and 70 'hits'.

Question: 3.

Based on your experiment, what is the probability of your pen landing on a crack?

Answer to Q2 ÷ 100.

Question: 4.

Simulate another 100 hits and record the results. Compare this set of results with your previous trial.

Answers will vary. Comments should reference that these outcomes represents 'trials' and therefore the results are expected to vary.

Question: 5.

You have completed 200 trials so far. Record your results for the 200 trials and 4 other friend's total results. (A total of 1,000 trials).

Use the 1,000 trials to estimate the probability that your pen would lie over a crack when dropped.

Answers will vary typically between 600 and 670 for the results of 1000 trials in total.

Navigate to page 2.1.

A program to simulate the dropping of the pen will automatically check the results of 100's trials. To access the program press the [**VAR**] key and select: **Buffon**.

Press [**Enter**] to run the program and enter the number of trials.
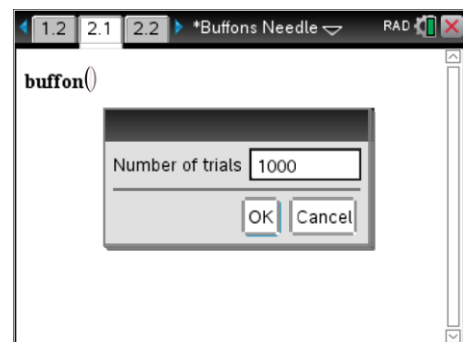
Question: 6.

Run the program 10 times completing 1000 trials in each. Record the results for your 10 trails (10,000 pen drop simulations in total). Use this data to record the probability of your pen landing over a crack in 10,000 trials.

Answers will vary, as above typically the 1000 trials yields results between 600 and 670.

The purpose of the question is to show that there is continued variability between trials despite the relatively large numbers of trials. The increase in the number of trials however continues to provide a better estimate for the actual probability.

Teacher Notes: *The results 'averages' of each set of 1000 trials could be plotted as a dot plot producing a 'sampling distribution' and subsequent normal distribution.*

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TEXAS INSTRUMENTS

Question: 7.

Get results from 9 other friends for their 10,000 trials and record the result in a table. Use all of these results to again estimate the probability of your pen landing on a crack.

This brings the total number of trials to 100,000 therefore providing a much improved estimate of the actual probability.

Question: 8.

Calculate the 'reciprocal' of your answer to Question 7, ("Flip it") and double the result. To what number is this similar? Is this a coincidence? – Discuss.

Doubling the reciprocal of the answer ≈ 3.1416

Teacher Notes: This is not a coincidence. For an advanced class a few simple diagrams will illustrate that the answer has something to do with 'circles' and 'trigonometry' … so therefore it is not surprising that the answer has something to do with π.
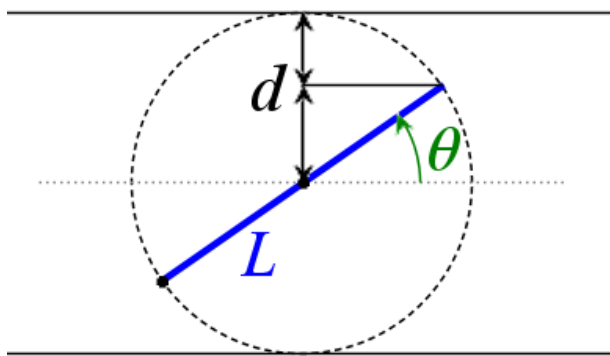
Let L = Length of 'pen'

The two lines are separated by length L.

Let d = distance from centre of pen to either line

Let θ = Angle between pen and line

Pen will not cross over either line if:

$$d > \frac{L}{2}\sin(\theta)$$

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TEXAS INSTRUMENTS