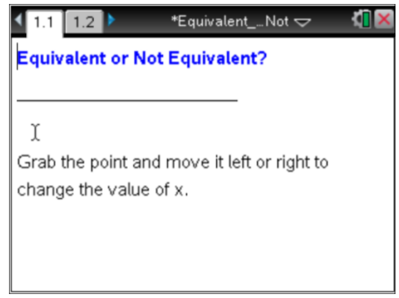


Math Objectives

- Students will identify equivalent expressions.
- Students will determine conditions of the variable under which expressions are or are not equivalent.
- Students will use appropriate tools strategically (CCSS Mathematical Practice).

Vocabulary

- equivalent expressions

About the Lesson

- This lesson will introduce the idea of equivalent expressions in the context of three critical operations (division, squaring, and absolute value) where what you "see" in the original expressions is not the mathematical story.
- The activity involves expressions that appear equivalent but are not, in fact, equivalent for all real numbers. The activity goes beyond having students recognize equivalent expressions to helping them understand that expressions may be equivalent for some, but not all, values of the variable. They will also recognize that the definition of an operation within an expression, such as absolute value, must be considered in making judgments about equivalence.
- Students will change the value of the variable *x* and observe the value of three different expressions. By comparing these values, students will identify which expressions are equivalent and which are not. They will recognize that the mathematical operations can be used to investigate whether the representations are actually equivalent.

TI-Nspire™ Navigator™ System

- Use Quick Poll to check student understanding.
- Use Screen Capture to examine patterns that emerge.
- Use Live Presenter to engage and focus students.

TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Tech Tips:

- Make sure the font size on your TI-Nspire handhelds is set to Medium.
- You can hide the function entry line by pressing ctrl G.

Lesson Materials:

Student Activity
Equivalent_or_Not_Student.pdf
Equivalent_or_Not_Student.doc

TI-Nspire document
Equivalent_or_Not.tns

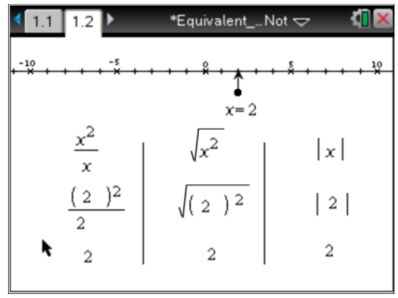
Visit www.mathnspired.com for lesson updates and tech tip videos.

Discussion Points and Possible Answers

Tech Tip: If students experience difficulty dragging a point, check to make sure that they have moved the cursor until it becomes a hand (👆) getting ready to grab the point. Also, be sure that the word point appears, not the word text. Then press ctrl 👆 to grab the point and close the hand (✊).

Move to page 1.2.

As a result of this problem, students should be able to understand the definition of equivalent expressions and recognize when equivalent expressions exist.



1. Find the value for each expression when

   a. $x = 2$. $\frac{x^2}{x}$ = _____ $\sqrt{x^2}$ = _____ $|x|$ = _____

   Answer: $\frac{x^2}{x} = 2$ $\sqrt{x^2} = 2$ $|x| = 2$

   b. $x = 4$. $\frac{x^2}{x}$ = _____ $\sqrt{x^2}$ = _____ $|x|$ = _____

   Answer: $\frac{x^2}{x} = 4$ $\sqrt{x^2} = 4$ $|x| = 4$

2. Based on your answers from question 1, predict the value for each expression when $x = 15$.

   $\frac{x^2}{x}$ = _____ $\sqrt{x^2}$ = _____ $|x|$ = _____

   Answer: $\frac{x^2}{x} = 15$ $\sqrt{x^2} = 15$ $|x| = 15$

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 1 at the end of this lesson.

3. Find the value for each expression when

a. x = –3. $\frac{x^2}{x}$ = _____ $\sqrt{x^2}$ = _____ |*x*| = _____

Answer: $\frac{x^2}{x}$ = –3 $\sqrt{x^2}$ = 3 |*x*| = 3

b. x = –8. $\frac{x^2}{x}$ = _____ $\sqrt{x^2}$ =_____ |*x*| =_____

Answer: $\frac{x^2}{x}$ = –8 $\sqrt{x^2}$ = 8 |*x*| = 8

4. Based on your answers from question 3, predict the value for each expression when *x* = –20.

$\frac{x^2}{x}$ = _____ $\sqrt{x^2}$ = _____ |*x*| = _____

Answer: $\frac{x^2}{x}$ = –20 $\sqrt{x^2}$ = 20 |*x*| = 20

Teacher Tip: It is important to have students share their thinking about these representations. Students should justify their responses based on more than the numerical examples by using the mathematical operations identified in the expressions. Be sure that students understand that the square root of a number is defined to be the non-negative root of the number. Thus $\sqrt{9}$ = 3. The only way to get the negative root is to multiply the root by –1, as in –$\sqrt{9}$. Both roots are solutions of the equation $x^2 = 9$, where $\sqrt{x^2}$ = ± $\sqrt{9}$, or *x* = ± 3. This concept will be explored further in questions 7 and 8. Students might also forget that the absolute value of a number can be thought of as the distance of the number from 0, and distance is always positive.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 2 at the end of this lesson.

5. Find the value for each expression when $x = 0$.

   a. $\frac{x^2}{x}$

   Answer: The value is undefined.

   b. $\sqrt{x^2}$

   Answer: 0

   c. $|x|$

   Answer: 0

6. a. Is the expression $\frac{x^2}{x}$ equivalent to x for the set of positive real numbers? Why or why not?

   Answer: Students should respond that these two expressions are equivalent because if you have the product of two x's and divide by one x, you get an x for the result. The mathematical justification involves the multiplicative inverse and identity principles: A number divided by itself is 1, and the product of 1 and any quantity is that quantity. Have students consider why it is true and how it relates to the question if the topic does not come up in the discussion.

   > **Teacher Tip:** The explanation above using factors is important for future work in algebra: $\frac{x \cdot x}{x} = x$ occurs in many different settings and should be well understood by students. The fact that a quantity divided by itself is equal to 1 is the reason that you can reduce arithmetic fractions as well as algebraic fractions. In reverse, it is the reason why you can obtain common denominators to be able to add fractions.

   b. Is the expression $\frac{x^2}{x}$ equivalent to x for the set of negative real numbers? Why or why not?

   Answer: Students should respond in the same way they did for problem 6a: These two expressions are equivalent because if you have two *x*s and divide by one *x*, you get an *x* for the result. The mathematics involves the multiplicative inverse and identity principles: A number divided by itself is one, and the product of one and any quantity is that quantity.

c. Is the expression $\frac{x^2}{x}$ equivalent to *x* for the set of real numbers? Why or why not?

Answer: The two expressions are not equivalent for all real numbers. $\frac{x^2}{x}$ is undefined when $x = 0$.

7. Tom says that the expression $\sqrt{x^2}$ is equivalent to *x* for the set of real numbers. Do you agree? Why or why not?

Answer: The set of real numbers includes all positive and negative rational numbers and 0. Students should recognize that the two expressions are not equivalent over both positive and negative numbers. For positive numbers, the statement is true, but it is not true for negative numbers. The radical is asking only for the positive root of the square of *x,* while x can represent either a positive or a negative number.

> **Teacher Tip:** Refer to the notes in prior problems to help students sort out the difference between the positive root and the negative root of a radical. Ask students to give examples when the statement is true and when it is not, and then encourage them to generalize.

8. For what values of x are $\sqrt{x^2}$ and $|x|$ equivalent? Explain your reasoning.

Answer: These expressions are equivalent for all real number values of *x* because you are taking the positive square root in one case and finding the distance from 0 in the other (the absolute value), which are both positive.

> **Teacher Tip:** While examples may support the way students' reason, it is important to help students understand why different replacement sets for the values of x result in different outcomes. In other words, the definitions of the operations or rules are critical. Ask them what the definition of absolute value means (the distance a number is from 0). How is the square root of a non-negative numerical value defined (the non-negative root of the number, the non-negative number whose square is the given product)? How do these definitions help you reason about the question?

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 3 at the end of this lesson.

Wrap Up

Upon completion of this discussion, the teacher should ensure that students are able to:

- Understand the importance of variable values in determining equivalence of expressions.
- Explain when $-x$ might be positive.
- Know when expressions are and are not equivalent.

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Note 1

Question 2, *Quick Poll*: Using Open Response, have students submit the three answers to question 2, separated by commas.

Note 2

Question 4, *Quick Poll*: Using Open Response, have students submit the three answers to question 4, separated by commas.

Note 3

Question 8, *Quick Poll*: Have students answer the following questions using Always, Sometimes, or Never:

1. $\sqrt{x} = x$ **Answer:** Sometimes. It is true when $x = 0$ or $x = 1$.
2. $|x| = x$ **Answer:** Sometimes. It is true when $x \geq 0$.

Have students answer the following question using True or False:

3. $|x| = -x$, when $x < 0$. **Answer:** True.