Induction for Tetrahedral Numbers

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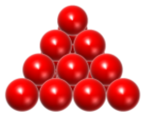
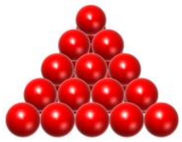
Student Activity

7 8 9 10 **11** 12

TI-Nspire™ | Investigation | Student | 50 min

Introduction

The purpose of this activity is to use exploration and observation to establish a rule for the sum of the first *n* tetrahedral numbers then use proof by mathematical induction to show that the rule is true for all whole numbers. What are the Tetrahedral numbers? The prefix 'tetra' refers to the quantity four, so it is not surprising that a tetrahedron consists of four faces, each face is a triangle. This triangular formation can sometimes be found in stacks of objects. The series of diagrams below shows the progression from one layer to the next for a stack of spheres.

Row Number	1	2	3	4	5
Items Added					
Complete Stack					

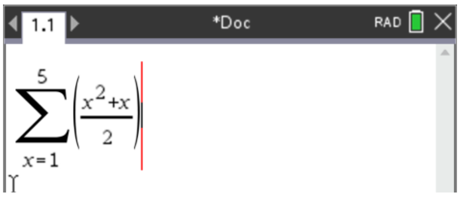
Question: 1.

Create a table of values for the row number and the corresponding quantity of items that are added to the stack.

Question: 2.

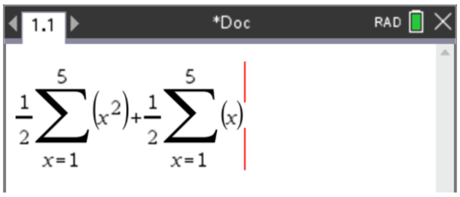
Create a table of values for the row number and the corresponding quantity of items in a complete stack.

Question: 3.



The calculator screen shot shown here illustrates how to determine the fifth tetrahedral number. The same command could be used to determine any of the tetrahedral numbers.
Explain how this command is working.

Question: 4.



Verify that the calculation shown opposite is the same as the one generated in Question 3.

Question: 5.

Enter the numbers 1, 2 … 10 in List 1 on the calculator. Enter the first 10 tetrahedral numbers in List 2. Once the values have been entered try the following:

a) Quadratic regression using List 1 and List 2. Check the validity of the result via substitution.

b) Cubic regression using List 1 and List 2. Check the validity of the result via substitution.

Authors: Peter Fox

Pascal's Triangle – Another Gem

Pascal's triangle also contains the tetrahedral numbers.

Example: The number 20 is the 4th tetrahedral number. It is located in the 6th row.

Recall that the elements in Pascal's triangle can be computed using combinatorics: ${}^nC_r = \dfrac{n!}{(n-r)!r!}$

							1							
						1		1						
					1		2		1					
				1		3		3		1				
			1		4		6		4		1			
		1		5		10		10		5		1		
	1		6		15		20		15		6		1	
1		7		21		35		35		21		7		1

Question: 6.

Use combinatorics to determine the value of the 100th Tetrahedral number. Check your answer using the equation established in the previous question and the summation tool on the calculator.

Question: 7.

Use Pascal's triangle to determine a formula for the Tetrahedral numbers.

Question: 8.

Given that the Tetrahedral numbers can be computed using:

$$\sum_{n=1}^{x}\left(\frac{n^2+n}{2}\right)$$

Use mathematical induction to prove that this is equal to: $\dfrac{x(x+1)(x+2)}{6}$.

Question: 9.

Question 4 used the property that $\displaystyle\sum_{n=1}^{x}\frac{n^2+n}{2}=\frac{1}{2}\sum_{n=1}^{x}n^2+\frac{1}{2}\sum_{n=1}^{x}n$.

Use this to show that: $\displaystyle\sum_{n=1}^{x}n^2=\frac{x(x+1)(2x+1)}{6}$

Question: 10.

Use mathematical induction to prove that: $\displaystyle\sum_{n=1}^{x}n^2=\frac{x(x+1)(2x+1)}{6}$

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