

Nonlinear Systems of Equations

ID: 9981

 Time required
 45 minutes

Activity Overview

This activity is designed to be used as an introduction to nonlinear systems of equations. It begins by allowing students to see the many ways that certain types of graphs (linear/conic and conic/conic) can intersect each other and how many potential intersection points are possible. Graphs include lines, parabolas, hyperbolas, ellipses, and circles. The activity concludes by having students look at the equations in a nonlinear system, stating how many solutions are possible, and then solving the system by graphing.

Topic: Conics

- *Verify the solution to a linear-quadratic system of equations by graphing and using Intersection Point(s).*

Teacher Preparation and Notes

- *This activity is designed to be used in a Precalculus classroom. It can also be used in an Algebra 2 classroom.*
- *This activity is intended to be mainly **teacher-led** with breaks for individual student work. Use the following pages to present the material to the class and encourage discussion. Students will follow with their calculators.*
- *Students should be familiar with the graphs and equations of the four conic section types: parabolas, circles, ellipses, and hyperbolas.*
- *Students should also understand transformations of parabolas, specifically how the sign of the squared term indicates whether the parabola opens up or down, as well as how to translate a parabola vertically.*
- *Students should also be familiar with parabolas that open left or right and how to enter them in their calculators.*
- *Students should be familiar with graphing functions.*
- **To download the student worksheet, go to education.ti.com/exchange and enter "9981" in the keyword search box.**

Associated Materials

- *NonlinearSystems_Student.doc*

If needed, define *nonlinear system of equations*. The system is one in which at least one equation is nonlinear. As with a linear system, the solutions of the system correspond to the intersection points of the graphs in the system.

Problem 1 – Number of Possible Intersection Points

Students look at the circle $x^2 + y^2 = 16$ and a line. They are to answer the question, *how many ways can a line intersect the circle?*

To draw the circle, they are to press **[2nd] [DRAW]** and select **9:Circle(**. Key in **Circle(0, 0, 4)** and press **[ENTER]**.

To draw a horizontal line, students can press **[2nd] [DRAW]** and select **3:Horizontal**, press **[ENTER]**. Using **[↑]** or **[↓]**, the line will move up or down. To draw a vertical line, students select **4:Vertical**, press **[ENTER]**. Using **[←]** or **[→]**, will move the line left or right.

A circle and line have 0, 1, or 2 intersection points.

Students will now look at the parabola $y = x^2 + 3$ and a line. *How many intersection points are possible between a line and a parabola?*

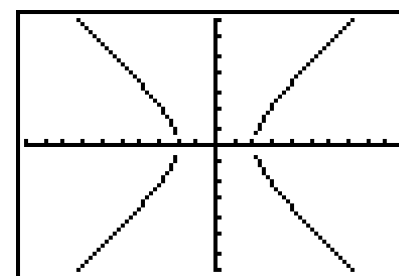
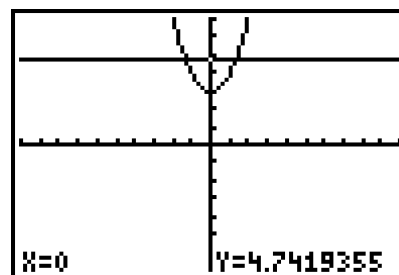
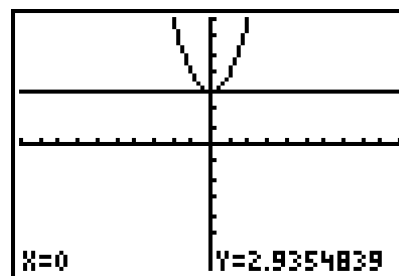
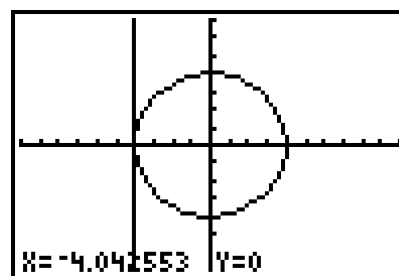
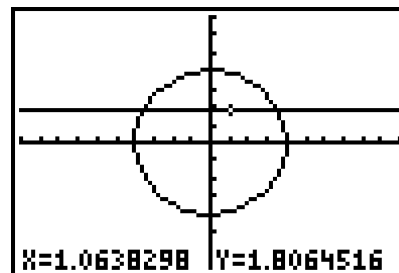
When students draw a horizontal line, they will get 0, 1, and 2 points of intersection. If the students use a vertical line, they will only get 0 or 1 point of intersection. Have them explain why this happens. A line on an angle will give 0, 1, or 2 possibilities as well.

The students should conjecture that for graphs of a linear and quadratic function, there are either 0, 1, or 2 points of intersection.

Students can test their conjecture with an ellipse and a hyperbola. They are to graph each of the functions with a horizontal and/or vertical line. They should confirm 0, 1, and 2 points of intersection.

Ellipse: $\frac{x^2}{25} + \frac{y^2}{16} = 1 \rightarrow y = \pm 4\sqrt{1 - \frac{x^2}{25}}$

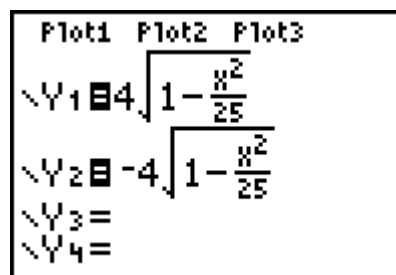
Hyperbola: $\frac{x^2}{4} - \frac{y^2}{4} = 1 \rightarrow y = \pm 2\sqrt{\frac{x^2}{4} - 1}$



If using Mathprint OS:

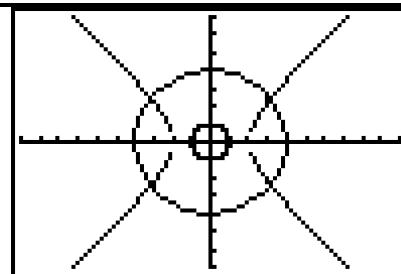
When entering the functions in Y1 and Y2, and students press [2nd] [√], the cursor will move under the radical sign. Students should enter the value of the radicand and then press [▢] to move out of the radical sign.

Note: Press [ALPHA] [F1] and select **n/d**, to insert the fraction template.



Students will now look at a hyperbola and a circle. *How many intersection points are possible between a hyperbola and a circle?*

Students are to change the radius and center of the circle. They should see that there are 0, 1, 2, 3, and 4 possible intersection points. The following are possible commands for constructing circles for these intersections.



Zero points: Circle(0, 0, 1)

Three points: Circle (-1, 0, 3)

One point: Circle (1, 0, 1)

Four points: Circle (0, 0, 4)

Two points: Circle (0, 0, 2)

Students should use the **CirDraw** command after each drawing of the circle.

Students will now look at a hyperbola and a parabola. *How many intersection points are possible between a hyperbola and a parabola?*

If students use the parabola $y = x^2$, they see that there is no intersection in the graph.

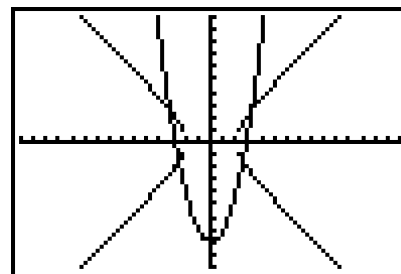
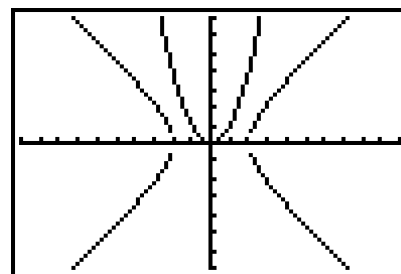
If they use the parabola $y = x^2 - 8$, they have four points of intersection in the graph.

Students should also find 1, 2, or 3 points by manipulating the parabola. Possible parabolas follow.

One point: $y = 4(x - 1)^2 - 4$.

Two points: $y = 4x^2 - 16$

Three points: $y = (x - 1)^2 - 9$



Students are to make a conjecture about the number of intersections between the graphs of two conic sections (there can be 0, 1, 2, 3, or 4 intersection points.)

Problem 2 – Two Parabolas

Direct students to make two different graphs that show two parabolas not intersecting. Two samples follow.

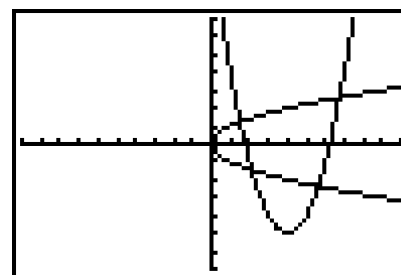
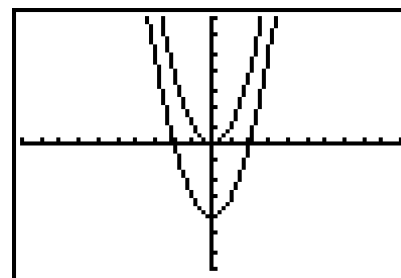
$$y = x^2 - 4 \text{ and } y = x^2$$

$$y = x^2 \text{ and } y = -x^2 - 1$$

Allow students time to work independently on making parabolas intersect in exactly 1, 2, 3, and 4 places. Remind the students that parabolas can open to the right or to the left as well. Also, remind students that they must define two functions to graph a parabola that opens to the right or to the left because that relation is not a function.

Sample parabolas follow. There are many more that can be done for each number of intersections.

- 1 point: $y = x^2$ and $y = -x^2$
- 2 points: $y = x^2$ and $y = 4 - x^2$
- 3 points: $y = \pm\sqrt{x}$ and $y = (x - 4)^2 - 2$
- 4 points: $y = \pm\sqrt{x}$ and $y = (x - 4)^2 - 5$.



Problem 3 – Solving Nonlinear Systems by Graphing

This problem consists of four systems of equations to solve. For each, students should first decide how many solutions are possible. Then, they find the approximate solutions by graphing the equations and finding the intersection points (**2nd** [CALC] and select **5:Intersect**).

Students can also solve the systems algebraically by using elimination or substitution.

$$\begin{cases} x^2 + y^2 = 25 \\ 2x^2 + 6y^2 = 18 \end{cases}$$

$$x^2 + y^2 = 25 \rightarrow y = \pm\sqrt{25 - x^2}$$

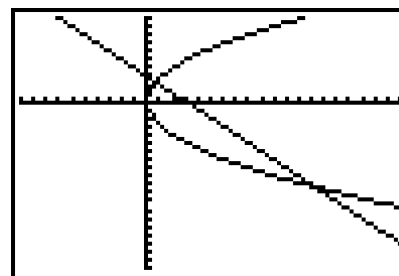
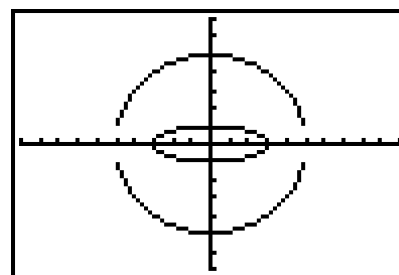
$$2x^2 + 6y^2 = 18 \rightarrow y = \pm\frac{\sqrt{9 - x^2}}{3}$$

No points of intersection.

$$\begin{cases} x + y = 3 \\ y^2 - 8x = 0 \end{cases} \quad [-10, 20] \text{ for } x, [-20, 10] \text{ for } y$$

$$y^2 - 8x = 0 \rightarrow y = \pm\sqrt{8x}$$

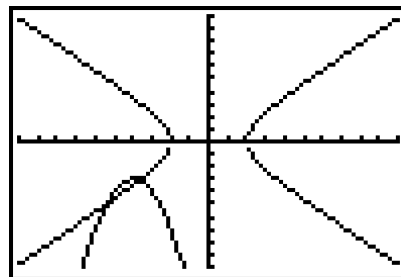
Two points: (0.68, 2.32) and (13.32, -10.32)



$$\begin{cases} x^2 - y^2 = 4 \\ y + x^2 = -8x - 19 \end{cases} \quad [-10, 10] \text{ for both } x \text{ and } y$$

$$x^2 - y^2 = 4 \rightarrow y = \pm\sqrt{x^2 - 4}$$

Two points: $(-3.69, -3.10)$ and $(-5.43, -5.05)$



$$\begin{cases} x^2 + y^2 = 15 \\ y + 6 = x^2 \end{cases} \quad [-10, 10] \text{ for both } x \text{ and } y$$

$$x^2 + y^2 = 15 \rightarrow y = \pm\sqrt{15 - x^2}$$

Four points: $(2.92, 2.54)$, $(-2.92, 2.54)$,
 $(1.57, -3.54)$, $(-1.57, -3.54)$

