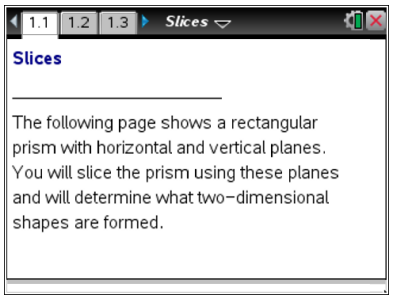
Slices

MATH NSPIRED

TEACHER NOTES

Math Objectives

- Students will predict and identify the shapes of two-dimensional cross-sections that are formed when three-dimensional figures are cut by horizontal and vertical planes.
- Students will draw, construct, and describe geometrical figures and describe the relationships among them.



Vocabulary

- rectangular prism
- triangular prism
- cross-section
- tetrahedron
- cylinder
- cone

About the Lesson

- This lesson involves exploring the shapes formed from 3D figures cut by a horizontal and/or a vertical plane.
- As a result, students will:
  - Predict the outcomes of moving planes through 3D figures.
  - Move a horizontal and vertical plane through the figure to check their predictions.
  - Explain any similarities or differences they see in the shapes that are formed as the planes move through the figure.
  - Determine what 3D figure was sliced given a description of the shapes formed from slicing the figure both horizontally and vertically.

TI-Nspire™ Navigator™

- Send out the *Slices.tns* file.
- Monitor student progress using Class Capture.
- Use Live Presenter to spotlight student answers..

Activity Materials

- Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ App for iPad®, TI-Nspire™ Software

Tech Tips:

- This activity includes screen captures taken from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:

Student Activity

- Slices_Student.pdf
- Slices_Student.doc

TI-Nspire document

- Slices.tns

Discussion Points and Possible Answers

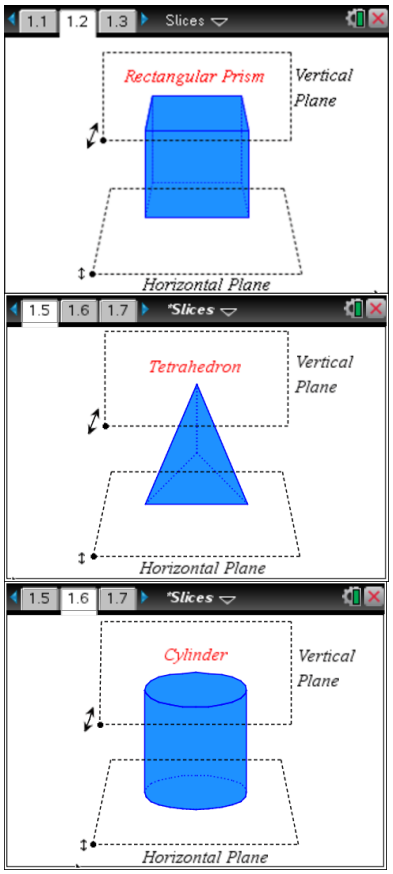
Before students move the two-dimensional planes through the three-dimensional figures to slice them, have students first make a prediction and fill in the table on their student activity sheets as described in question 1. Students may need to draw multiple sketches if they believe the slices will change as the plane cuts through the figure from one end to the other.

Tech Tip: If students experience difficulty dragging the point for the planes, check to make sure that the cursor becomes hand (). Then be sure that they select ctrl to close the hand ().

Tech Tip: To move the two-dimensional planes through the three-dimensional figures, grab a point near a double-ended arrow and drag it.

Before students begin they are instructed first make a prediction and fill in the table on the next page. If they think the shape will change as the two-dimensional plane slices the figure, they are to draw multiple sketches that show how the shape changes as the plane slices from one end of the figure to the other. Then they are to slice the three-dimensional figures using the TI-Nspire pages.

Move to pages 1.2 through 1.7.

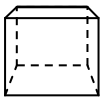


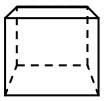
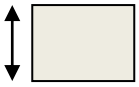
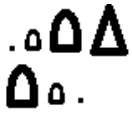
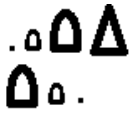
1. For each of the 3-dimensional figures on pages 1.2-1.7:
   a. Make sketches for your predictions of what two-dimensional shapes will appear if you were to cut the figure **horizontally** from the bottom to the top. Make your sketch in the table provided below.
   b. When you have made your predictions, check your answers by actually sliding the horizontal plane through the figure on each page. (When you are finished, be sure to return the horizontal plane to the bottom of the figure.)
   c. Make sketches for your predictions of what two-dimensional shapes will appear if you were to cut the figure **vertically** from the back to the front. Make your sketch in the table provided below.
   d. When you have made your predictions, check your answers by actually sliding the vertical plane through the figure on each page.
   e. After you have drawn the sketches for the actual two-dimensional shapes, note how the two-dimensional shape changes as you move the planes through the figures.

Teacher Tip: Have students show some of their predictions to their neighbor and discuss why they drew what they did. Be sure that students use precise descriptions to explain their predictions. For example, it is not precise enough to say "the shape will change from a small circle to a big circle." Students must precisely describe whether they are referring to the horizontal or vertical cross-section, and they should mention whether they are "slicing" it from top to bottom, back to front, etc.

Teacher Tip: You might want to model the cross-sectional concept with the first 3D figure. If possible, have some actual 3D models available for students to hold. Some students may think the horizontal slices create trapezoids from the rectangular prism because of the perspective from which they are viewed. You may need to have a class discussion about the distortion issues for each figure. For example, the slices of the rectangular prism appear to be different sizes due to perspective, but they really should be the same size.

Sample Answers: See answers in the table below, but students' answers will vary (especially between their predictions and the actual answers).

		Rectangular Prism	Triangular Prism	Square Pyramid	Tetra-hedron	Cylinder	Cone
							
Horizontal Slicing	**Sketches of Horizontal Slice Predictions** 						
	Sketches of Actual Horizontal Shapes						
	What changes?	Nothing.	Nothing.	As you move bottom to top, slices are squares that decrease in size and eventually turn to a point.	As you move bottom to top, slices are triangles that decrease in size and eventually turn to a point.	Nothing.	As you move bottom to top, slices are circles that decrease in size and eventually turn to a point.
	What stays the same?	The dimensions of the rectangle are always the same.	The dimensions of the triangle are always the same.	The slices are squares until you reach the top.	The slices are triangles until you reach the top.	The dimensions of the circle are always the same.	The slices are all circles until you reach the top.

		Rectangular Prism	Triangular Prism	Square Pyramid	Tetra-hedron	Cylinder	Cone
							
Vertical Slicing	**Sketches of Vertical Slice <u>Predictions</u>** 						
	Sketches of <u>Actual</u> Vertical Shapes						
	What changes?	Nothing.	As you move back to front, slices turn from a line segment to rectangles that increase in size.	As you move back to front, slices turn from a line segment to trapezoids that increase in size to a triangle when you reach the apex and then they turn back to trapezoids that decrease in size and finally to a line again.	As you move back to front, slices turn from a line segment to triangles that increase in size to trapezoids that decrease in size and finally to a line again.	As you move back to front, slices turn from a line segment to rectangles that increase in size to rectangles that decrease in size and finally to a line again.	Moving from back to front, slices turn from a point to shapes with a flat bottom but a curved top that increase in size to a triangle at the apex and then back to shapes with a curved top that decrease in size and finally to a point again.
	What stays the same?	The dimensions of the rectangle are always the same.	Nothing.	Nothing.	Nothing.	Nothing.	Nothing.

TI-Nspire Navigator Opportunity: *Class Capture and Live Presenter*
See Note 1 at the end of this lesson.

2. For which of the six figures did you obtain:
   a. Rectangular shapes by slicing? If there is more than one figure, how are these figures related?

      Answer: The rectangular prism, triangular prism, and the cylinder produce rectangular shapes. These figures are related because they all have a pair of parallel bases.

   b. Triangular shapes by slicing? If there is more than one figure, how are these figures related?

      Answer: The square pyramid, tetrahedron, and the cone produce triangular shapes. These figures are related because they all have only one base.

   c. Circular shapes by slicing? If there is more than one figure, how are these figures related?

      Answer: The cylinder and the cone produce circular shapes. These figures are related because they both have at least one circular base.

   d. Trapezoidal shapes by slicing? If there is more than one figure, how are these figures related?

      Answer: The tetrahedron and the square pyramid produce trapezoidal shapes. These figures are related because they both have triangular lateral faces.

   e. A line by slicing? If there is more than one figure, how are these figures related?

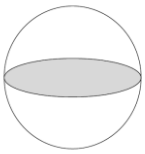
      Answer: The triangular prism, the square pyramid, and the tetrahedron all produce lines. These figures are related because they all have at least one base that is a polygon.

   f. A point by slicing? If there is more than one figure, how are these figures related?

      Answer: The square pyramid, the tetrahedron, and the cone produce points. These figures are related because they all have only one base.

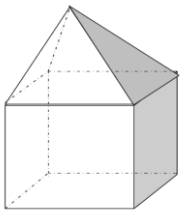
TI-Nspire Navigator Opportunity: *Quick Poll and Live Presenter*
See Note 2 at the end of this lesson.

3. When sliced vertically back to front and horizontally bottom up, a particular three-dimensional figure first produces a point, then circles that increase in size, then circles that decrease in size, and then finally a point again. What would that three-dimensional figure be? Draw a sketch.



Answer: It is a sphere.

4. When sliced vertically from back to front, a composite three-dimensional figure first produces a square, then hexagons that increase in size, then a pentagon, then hexagons that decrease in size, and then finally a square. When sliced horizontally from the bottom up it produces squares that are first the same size and then eventually squares that decrease in size to a point. What would that three-dimensional figure be composed of? Draw a sketch.



Answer: It is a cube with a square pyramid on top.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 3 at the end of this lesson.

Extensions

- For each 3D figure and for both the horizontal and vertical planes, have students determine when the shape produced has the largest area. For example, in the case of the square pyramid and the vertical plane, it would occur when the plane reached the apex of the pyramid.
- For each 3D figure, have students determine what two-dimensional (2D) figures would be produced if the plane were oblique (slanted) instead of horizontal or vertical.

Wrap Up

Upon completion of the lesson, the teacher should ensure that students are able to understand:

- Various two-dimensional figures result from slicing three-dimensional figures with horizontal and vertical planes.

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Note 1

Question 1, Class Capture and Live Presenter

Take Class Captures as students go through Pages 1.3 – 1.7. As a class, discuss the various cases that occur. Allow a student (or students) to show the various two-dimensional shapes that appear as they slice the three-dimensional figures.

Note 2

Question 2, Quick Poll and Live Presenter

Send a Quick Poll question for each part to determine student responses to Questions 2a–f. As a class, discuss the results. Allow a student (or students) to justify their responses to Questions 2a–f. As a class, discuss the results.

Note 3

Question 3, Quick Poll

Send a Quick Poll question for each part to determine student responses to Questions 3 and 4. As a class, discuss the results.