Complex Number Addition
Student Activity

Name ________________
Class ________________

Open the TI-Nspire document *Complex_Number_Addition.tns.*

In this activity, you will compute, visualize, and geometrically interpret the sum of two complex numbers such as $z = a + bi$, $w = c + di$, and the sum $s = z + w$.

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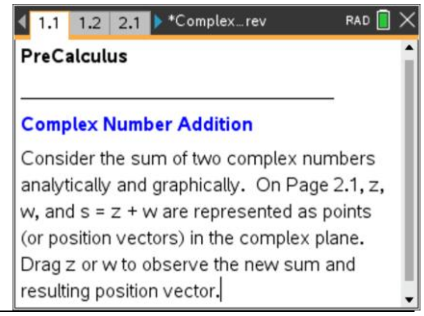

Move to page 1.2.

Press ctrl ▶ and ctrl ◀ to navigate through the lesson.

1. This Notes page contains three interactive Math Boxes for the complex numbers z, w, and the sum $s = z + w$.

   a. Redefine z and/or w as necessary to complete the following two tables. To redefine z or w, edit the Math Box following the assignment characters (i.e., $:=$).

z	$3+5i$	$-3-4i$	$11-11i$	$-5-6i$
w	$-4+7i$	$-2+6i$	$-11+12i$	$-7-9i$
$z+w$				

z	$-\frac{1}{2}-\frac{3}{4}i$	$1-\sqrt{2}i$	$\frac{\sqrt{3}}{2}-3i$	$\frac{3}{5}-\frac{4}{5}i$
w	$1+\frac{1}{4}i$	$-1-\sqrt{2}i$	$\frac{\sqrt{3}}{2}+3i$	$\frac{2}{5}-\frac{4}{5}i$
$z+w$				

   b. Let $z = a + bi$ and $w = c + di$. Explain in words how the complex numbers are added in terms of the real parts and the imaginary parts.

   c. Let $z = a + bi$ and $w = c + di$. Write the sum, $s = z + w$, symbolically in terms of the constants a, b, c, and d.

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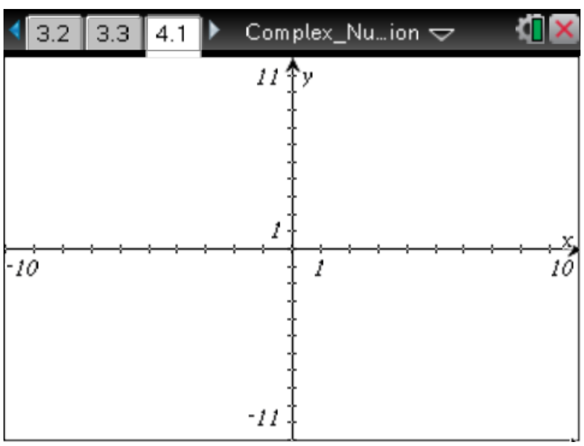
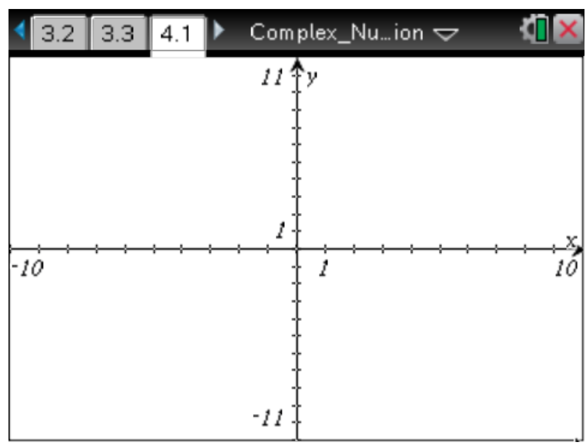
2. In the left panel, the complex numbers z and w are represented by points and position vectors in the plane. Point s represents the sum of these two complex numbers. Drag either point z or point w, and the sum is automatically computed and updated. The right panel displays the actual values for z, w, and s.
   a. Drag points z and w around the plane, and observe the results. Explain addition of complex numbers geometrically.

   b. Position point z in the second quadrant and point w in the first quadrant. On the first set of axes below, sketch a figure representing the resulting sum $s = z + w$. On the second set of axes below, sketch a figure that you think might represent the difference $d = z - w$. Drag and position point w to confirm your hypothesis. Hint: $d = z + (-w)$.



$s = z + w$



$d = z - w$

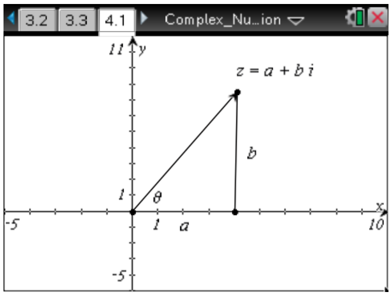
Move to page 3.1.

3. This page is a copy of Page 2.1 such that the real and imaginary parts of points z and w move only in increments of 0.5.
   a. Drag and position point z and/or point w so the sum is 0—that is, $s = 0 + 0i$ and is represented by a point at the origin. Explain the relationship between points z and w.

   b. Drag and position point z and point w such that $z = 2 + 2i$ and $w = 5 + 5i$. Find the sum s, and explain the relationship between the points representing z, w, and s.

c. The absolute value or magnitude of a complex number $z = a + bi$ is $|z| = \sqrt{a^2 + b^2}$. Find the absolute value of z, w, and s in part 3b, and explain how these three values are related.

The argument of a complex number $z = a + bi$ is the angle, θ, (in radians) formed between the positive real axis and the position vector representing z. See the figure to the right. The angle is positive if measured counterclockwise from the positive real axis. Recall, $\tan\theta = \frac{b}{a}$.



d. Describe a method to find the argument of the complex number z in part 3b above. Find the actual argument for z, w, and s in part 3b. Explain how these three arguments are related.

4. Drag and position point z and point w such that $z = 2 + 2i$ and $w = -5 - 5i$.

a. Find the sum s, and explain the relationship between the points representing z, w, and s.

b. Find the absolute value of z, w, and s in part 4a, and explain how these three values are related.

c. Find the argument of points z and w. How are they related?