Ball Sports

Teacher Notes and Answers

7 8 **9** **10** 11 12

TI-Nspire™ Investigation Student 50 min

Introduction

In this activity you will explore a range of ball-sports to see if a relationship exists across a range of sports when it comes to ball design.

Teacher Notes:

Students will need access to the Internet or a range of balls (Check with your PE department) to research ball size and mass.

There are a range of websites that provide data, for example:

https://www.topendsports.com/resources/equipment-ball-weight.htm

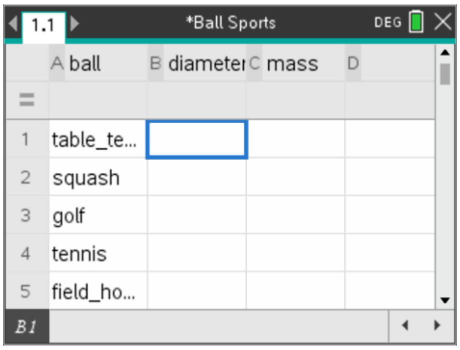
https://www.topendsports.com/resources/equipment-ball-size.htm

Data Collection

Open the TI-Nspire document: Ball Sports.

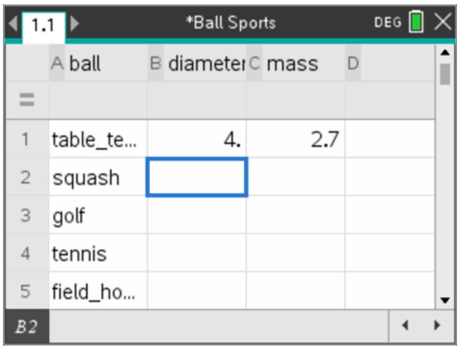
The document contains a list of balls used in common sports. Column B contains the heading: diameter and column C contains the heading: mass.

For this activity the diameter is measured in centimetres (cm) and mass in grams (g)



Your first task is to collect data for each of the ball types. Do not enter units, just the measurements. The table tennis ball is shown opposite.

Once the data has been entered, construct a scatterplot of ball diameter (independent or explanatory variable) on the horizontal axis and mass (dependent or response variable) on the vertical axis.



Question: 1

A cricket ball is not included in the original list. If cricket ball data was added to the list, where would it reside on the scatterplot?

Answer: A cricket ball is approximately the same size as a tennis ball but significantly heavier, it would therefore sit considerably higher on the vertical axis.

Question: 2

Use the Analysis option to create a linear regression model for the data and determine the equation.

a) What is the equation for the least squares regression line?

Answer: The equation will vary slightly, depending on student data: mass ≈ 24 x diameter – 60. ($r^2 \approx 0.95$)

Author: P. Fox

b) A new ball sport is being developed. The proposed ball has a diameter of 14cm. Determine an appropriate mass for the ball, assuming it falls into line with other ball sports.

Answer: Answers will vary slightly. Students need to substitute 14 into their equation. mass ≈ 276 grams

c) A new children's size basketball is being developed. It is made from the same material as the sanctioned basketball but is made lighter so that it is easier for young children to throw. If the new ball is half the mass of the original, suggest a size for this new ball.

Answer: Answers will vary, students should use their least squares regression equation and the half the mass they obtained for the basketball (in their data) then solve the corresponding equation: diameter ≈ 15cm

Australian Rules Football

Unlike all the other balls in the data, an Australian Rules football approximates an ellipsoid, so it doesn't have a diameter. In this section you will explore where the Australian Rules football might fit amongst the data.

Question: 3

One way to compare would be to average the three measurements: height, width and length and substitute this as the 'diameter' measurement. Collect the appropriate data for an Australian Rules football and plot it on the scatterplot.

Answer: A size 5 – adult's football is approximately 28cm (Length) x 17.5cm (width) x 17.5cm (height). The mean of these three dimensions is: 21cm. An AFL football weighs approximately 470 grams. The least squares regression equation: mass ≈ 24 x diameter – 60, for a 21cm ball estimates a mass of 444 grams. This means an AFL ball is slightly heavier than predicted.

Note: The question refers to the 'average' of the three measurements. Whilst it doesn't make sense to use a median or mode, a geometric mean could be used: $\sqrt[3]{28\times 17.5\times 17.5}\approx 20.5$. The slightly smaller mean results in a slightly smaller prediction for the mass of the ball: 432 grams.

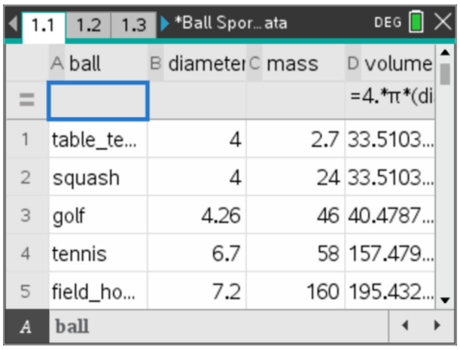
Teacher Notes: The slightly heavier nature of a football should not be surprising. An ellipsoid will have a larger surface area than a sphere of the same size, therefore it is likely to require more materials to produce the ball, therefore making it heavier.

Another way to compare an Australian Rules football would be to consider comparing all balls by volume.

Question: 4

Use column D in the spreadsheet to record the volume of each ball. **Remember**, column B contains the diameter of each ball. Record all your volume measurements.

The formula for the volume of a sphere is: $V=\dfrac{4}{3}\pi r^3$



	A ball	B diameter	C mass	D volume
=				=4.*π*(di
1	table_te...	4	2.7	33.5103...
2	squash	4	24	33.5103...
3	golf	4.26	46	40.4787...
4	tennis	6.7	58	157.479...
5	field_ho...	7.2	160	195.432...

Answer: Students can use a cell or column formula to automate the calculations. Students must remember to divide the diameter by 2 first in order to obtain the correct volume.

Students should be aware that they can simply copy and paste their results into a Word document, Spreadsheet or simply attach a copy of their TNS file.

Question: 5

Graph volume versus mass in the scatterplot. The linear regression line no longer fits the data nicely.

a) Express the volume in terms of the diameter of a sphere.

Answer: $V=\dfrac{4}{3}\pi\left(\dfrac{d}{2}\right)^3=\dfrac{\pi d^3}{6}$

Author: P. Fox

b) Transpose your volume equation to make D (diameter) the subject.

Answer: $d=\sqrt[3]{\dfrac{6V}{\pi}}$

c) Replace the D in your original least squares regression equation with your equation from (b), above.

Answer: $mass \approx 24\sqrt[3]{\dfrac{6V}{\pi}}-60$

d) The volume of an ellipsoid is given by the equation: $v=\dfrac{\pi hwd}{6}$ where h = height, w = width and d = depth.

Determine the volume of a football.

Answer: $v=\dfrac{\pi\times 28\times 17.5\times 17.5}{6}\approx 4489cm^3$

e) Use your equation from part (c) to estimate the mass of a football and comment on your answer.
Answer: 431 grams. The estimate is slightly lower than the actual mass of a football, but lies within a reasonable 'distance' of the regression equation prediction.
Teacher Notes: Students may notice the connection with the geometric mean through these questions.

Question: 6

Russell notices that ball density decreases with size. Use your volume vs mass relationship to show that Russell's statement is correct.
Answer: Density = mass ÷ volume, this is equivalent to the 'gradient' of the relationship. As the volume increases the gradient of our curve gets smaller.
Sample calculation: Density of a basketball: 0.087g/cm^3. Density of a squash ball: 0.72 g/cm^3.
This makes sense also, larger balls are generally filled with air and thin outer layer of material. (Leather/Rubber etc.)

Question: 7

Daisy is designing a new ball using a special rubber-polymer compound with a density of 0.8 g/cm^3. The lining of the ball is 1cm thick. To ensure the ball conforms approximately with the relationship between ball diameter and mass, how large should Daisy make the ball?
Answer: Answers will vary, however they should start with a calculation for the mass. The mass of the ball can be calculated by multiplying the volume (of the material) by the density.
Let r represent the radius of the ball. The inner radius is therefore $r-1$.

Volume of material required to make the ball: $V=\dfrac{4}{3}\pi r^3-\dfrac{4}{3}\pi(r-1)^3=\dfrac{4\pi(3r^2-3r+1)}{3}$

Mass of material required to make the ball: $m=\dfrac{16\pi(3r^2-3r+1)}{15}$

The original relationship between mass and diameter (Least Squares Regression): $M=24d-60$

Combining these last two equations and the fact that d = 2r, we get:

$48r-60=\dfrac{16\pi(3r^2-3r+1)}{15}$. Solving this equation produces two solutions: r = 1.46 or r = 4.31.
The smaller radius is too small, however the 4.31cm (diameter = 8.62) is between a tennis ball and baseball.

Author: P. Fox

Teacher Notes: There are many opportunities to customise this activity. For example: students can check the volume of a football by immersing it in a tub. Fill the tub with water, all the way to the top! Immerse the football in the water either fully (physically challenging whilst trying to keep your hands out of the water) or just half way. Water will overflow. Use a measuring jug to refill the tub making note of the volume of water poured back into the tub.

Author: P. Fox