The Painted Cube
Student Activity
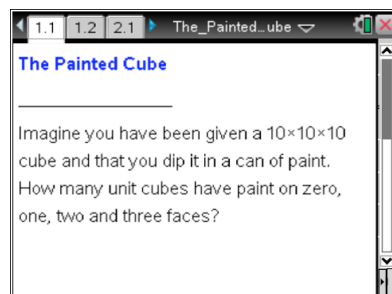
Name _____

Class _____

Open the TI-Nspire™ document The_Painted_Cube.

A cube with dimensions $n \times n \times n$ that is built from unit cubes is dipped in a can of paint. You will model the relationships between the dimensions of a cube and the number of faces of the unit cubes that are painted.

Part 1—Introduction to the Problem

Press `ctrl` ▶ and `ctrl` ◀ to navigate through the lesson.

Move to page 1.2.

One strategy for solving a problem is to solve a simpler related problem.

1. Consider a 2 × 2 × 2 cube.

   a. How many unit cubes does it take to build a 2 × 2 × 2 cube?

   b. Rotate the model on page 1.2 by dragging the open points on the left side of the screen. If needed, build your own model using cubes. If this cube were dipped in paint, what is the greatest number of faces of a single unit cube that could be painted?

   c. How many faces of *each* of the unit cubes are painted on the 2 × 2 × 2 cube?

Move to page 2.1.

2. Now consider a 3 × 3 × 3 cube.

   a. How many unit cubes does it take to build a 3 × 3 × 3 cube?

   b. Rotate the model on page 2.1 by dragging the open points on the left side of the screen. If needed, build your own model using cubes. If the 3 × 3 × 3 cube were dipped in a can of paint, how many faces of *each* of the unit cubes would be painted?

3. a. Record your findings for the 2 × 2 × 2 and 3 × 3 × 3 cubes in the table below. Then determine how many faces of each of the unit cubes would be painted for the 4 × 4 × 4 and 5 × 5 × 5 cubes if the large cubes were dipped in paint.

n (side length of cube)	Number of unit cubes with paint on zero faces	Number of unit cubes with paint on one face	Number of unit cubes with paint on two faces	Number of unit cubes with paint on three faces
2				
3				
4				
5				

b. What patterns do you notice in the table?

Part 2—Investigating Paint on Three Faces

Move to page 2.2.

You will now analyze the data you collected and explore the relationships graphically for a cube with any side length n. You will then use the graph to make predictions for the case where n = 10.

Move to page 3.1.

4. Enter the values from your table above into the spreadsheet on page 3.1.

5. From the information in the table, how many unit cubes would have paint on <u>three</u> faces in a 10 × 10 × 10 cube? Explain your reasoning.

Part 3—Investigating Paint on Two Faces

Move to page 3.2.

This page uses the data that you entered on page 3.1 to make a scatter plot of the number of unit cubes with paint on two faces versus the side length of the cube, *n*.

6. Describe the relationship between the two variables.

7. Add a movable line by selecting **Menu > Analyze > Add Movable Line**.

   a. Grab the line and transform it to get a line of best fit. What is the equation of your line of best fit?

   b. Test your equation with known values from your table and adjust your movable line as necessary. Once your equation matches the known values, what is the equation of your line?

   c. Write your equation in factored form. What is the meaning of this form of the equation in the context of the painted cube problem?

8. Use your equation to determine the number of unit cubes that would have paint on <u>two</u> faces in a 10 × 10 × 10 cube.

9. Explain how your answer makes sense in terms of the graph on page 3.2.

Part 4—Investigating Paint on One Face

Move to page 3.3.

This page uses the data that you entered on page 3.1 to make a scatter plot of the number of unit cubes with paint on one face versus the side length of the cube, *n*.

10. Describe the relationship between the two variables.

11. Determine the equation of the curve of best fit. Press **Menu > Analyze > Regression** and select the type of function that you think will best fit the data.

   a. What is the regression equation?

   b. Test your equation with known values from your table. If needed, choose a different type of regression equation. Once the equation matches the known values, what is the equation?

   c. Write your equation in factored form. What is the meaning of this form of the equation in the context of the painted cube problem?

12. Use your equation to determine the number of unit cubes that would have paint on <u>one</u> face in a 10 × 10 × 10 cube.

Part 5—Investigating Paint on Zero Faces

Move to page 3.4.

This page uses the data that you entered on page 3.1 to make a scatter plot of the number of unit cubes with paint on zero faces versus the side length of the cube, *n*.

13. Describe the relationship between the two variables.

14. Determine the equation of the curve of best fit. Press **Menu > Analyze > Regression** and select the type of function that you think will best fit the data.

   a. What is the regression equation?

   b. Test your equation with known values from your table. If needed, choose a different type of regression equation. Once the equation matches the known values, what is the equation?

15. Use your equation to determine the number of unit cubes that would have paint on <u>zero</u> faces in a 10 × 10 × 10 cube?

Part 6—Reflecting on the Problem

16. a.   Record the type of relationship (e.g., linear, quadratic) for each of the numbers of painted faces you investigated.

Painted Faces	Type of Relationship
3	
2	
1	
0	

b.   Think about the painted cubes and how the numbers of painted faces change as the side length of the cube, n, increases. Justify why each type of relationship makes sense in the context of the problem.