



Repeating Decimals and Fractions

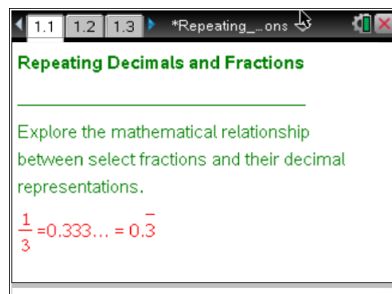
Student Activity

Name _____

Class _____

Open the TI-Nspire document *Repeating_Decimals_and_Fractions.tns*.

All rational numbers can be expressed as fractions or decimals with a finite number of digits or infinitely repeating digits. In this activity, you will explore the mathematical relationship between select fractions and their decimal representations.



Move to page 1.2.

Press **ctrl** **▶** and **ctrl** **◀** to navigate through the lesson.

1. First, explore fractions with a denominator of 9. Use the slider to change the numerator of the fraction, and observe the changes in the decimal representation of the fraction. Record your observations in the table.

Fraction	Decimal representation
$\frac{1}{9}$	
$\frac{2}{9}$	
$\frac{4}{9}$	

2. Describe any patterns that you observed.

3. Based on your observations predict the repeating decimals for the fractions given in the table below. Then, use the slider to check your predictions, and record the actual repeating decimal in the table:

Fraction	Predicted repeating decimal	Actual repeating decimal
$\frac{5}{9}$		
$\frac{6}{9}$		
$\frac{7}{9}$		
$\frac{8}{9}$		

4. What is the repeating digit in the decimal representation of the fraction $\frac{n}{9}$ when n ranges from 1 to 8?



5. Move the slider to find three fractions with decimal representation that do not have repeating digits. What do you notice about these fractions? Why does the decimal representation have no repeating digits?

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Next, explore repeating decimals, and find the fractions for those decimals. Notice that Page 1.3 is divided into three regions:

- The top work area provides directions for the Calculator App
- The Calculator App is in the lower left work area of the screen.
- The lower right work area contains the equivalent decimal and fraction for the number you will enter.

To use the Calculator App, that work area must be active. It is active when there is a dark border around it. If there is no dark border, press **ctrl** **tab** until the border becomes dark.

Follow the instructions below for entering decimals into the program:

- The decimal values entered must be between 0 and 1.
- The decimal must begin with "0."
- The decimal must not contain more than ten digits after the decimal point.
- The repeating digit (or group of digits) should be included at least three times followed by the ellipsis "..." to indicate the number is a repeating decimal.

6. In the table below, predict fractions for the given decimals with one repeating digit.

Decimal with one repeating digit	Predicted fraction	Actual fraction
0.22222...		
0.44444...		
0.66666...		
0.99999...		

7. Press **enter** to activate the program in the Calculator work area. In the pop-up window, input each decimal with one repeating digit, one at a time. Press OK. Record actual fractions in the table above.
8. Did any repeating decimal with one repeating digit result in a fraction with a denominator not equal to 9? Why?



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9. Now, let's explore fractions with denominator 99. Use the slider to change the numerator, and observe the changes in the decimal representation of the fraction. Record your observations in the table.

Fraction	Decimal representation
$\frac{1}{99}$	
$\frac{5}{99}$	
$\frac{10}{99}$	
$\frac{27}{99}$	

10. Based on your observations predict the repeating decimals for the fractions given in the table below. Then, use the slider to check your predictions, and record actual repeating decimals in the table.

Fraction	Predicted repeating decimal	Actual repeating decimal
$\frac{22}{99}$		
$\frac{23}{99}$		
$\frac{66}{99}$		
$\frac{98}{99}$		

11. Use the long division algorithm to compute $23 \div 99$. What do you notice about the remainders appearing in each step of division? Why do you get the two repeating digits when dividing 23 by 99?
12. Move the slider to find at least three fractions with decimal representations that have one repeating digit. What do you notice about these fractions?
13. Can you find a fraction that does not result in repeating decimal? If so, explain why there are no repeating digits.

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14. Using the program in Calculator App, find fractions for the decimals given in the table. Record your findings in the table.

Repeating decimal	Fraction
0.292929...	
0.343434...	
0.717171...	

15. What patterns do you observe for the decimals with two repeating digits?
16. Record several decimals with two repeating digits in the table below, and predict the equivalent fractions based on your earlier observations. Then check your predictions on Page 1.3.

Decimal with two repeating digits	Predicted fraction	Actual fraction

17. What is a common characteristic of fractions with decimal representation containing two repeating digits?
18. Now predict fractions for the given decimals with three repeating digits. Check to see if your predictions are correct. What do you notice about the decimals that have three repeating digits?

Decimal with three repeating digits	Predicted fraction	Actual Fraction
0.235235235...		
0.707707707...		
0.997997997...		

19. What is a common characteristic of fractions with decimal representation containing three repeating digits?



20. Explain how division might result in a decimal with a three repeating digits.

21. Record your predictions for the decimals given in the table below. Check to see if your predictions are correct. What do you notice about the fractions for these repeating decimals? Explain why the denominator is not equal to 999.

22. Can you find fractions with the denominator 999 that have only one repeating digit? What do you notice about these fractions?

23. Input decimals given in the table into the program on Page 1.3 and record the equivalent fractions. What do you notice about the denominators of these fractions? How can you predict the numerators?

Decimal	Actual fraction
0.1444...	
0.2111...	
0.4777...	

24. Based on your observations, record predicted fractions for the decimals given in the table below. Use the program on Page 1.3 to see if your predictions are correct. Explain the actual results.

Decimal	Predicted fraction	Actual fraction
0.1333...		
0.5111...		
0.3363636...		

25. Now record your predictions for a different set of decimals given below. Check to see if your predictions are correct. Explain how you can predict the numerator and denominators.

Decimal	Predicted fraction	Actual fraction
0.13222...		
0.08111...		
0.97444...		