

Jack's Tile Problem – Exploring the Area Rule for Triangles. Teacher Notes



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7 8 9 **10** 11 12



TI-Nspire CAS



Investigation



Student



50 min

Aim

The aim of this investigation is to use various triangles to explore, develop and apply the area rule.

National Curriculum Statement: Establish the sine, cosine and area rules for any triangle and solve related problems (ACMMG273)

ScOT: Trigonometry

Equipment

For this activity you will need:

- TI-Nspire CAS
- TI-Nspire CAS document – Trig Area Rule

Problem Description


The rules that you would have studied to date ($\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$, Pythagoras' Theorem and SOHCAHTOA) are useful for calculating unknown information about right-angled triangles and triangles given a perpendicular height. Unfortunately, not all situations involve such triangles. How can we find the area of triangles that are not right-angled or do not give a base and height measurement?

Technology

During this activity, students will need to use the TI-Nspire file: "Trig Area Rule". This file can be distributed using TI-Navigator, the TI-Nspire docking station or the teacher/student software. To distribute the file using the Teacher software, use the **Tools** menu and select the **Transfer Tool**. Locate the TI-Nspire file on your computer and then start the transfer. Once the file is transferred to the first handheld, unplug the handheld and continue plugging in each student's handheld device. Once all the students have the file, stop the transfer. Note that students can also transfer files from one handheld device to another from within the **My Documents** folder. Note also that multi-port USB connectors can be used to transfer files to several computers at the one time.

The activity requires access to the “Trig Area Rule” TI-Nspire document. This document should be loaded on your device before proceeding. You should also ensure your calculator mode is set to **Degree**.



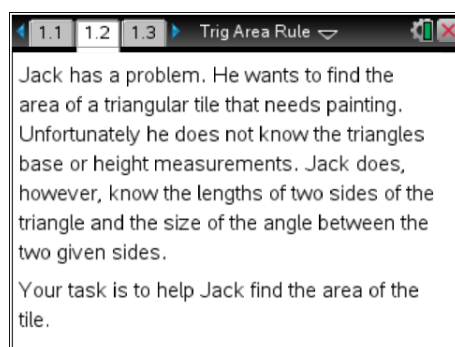
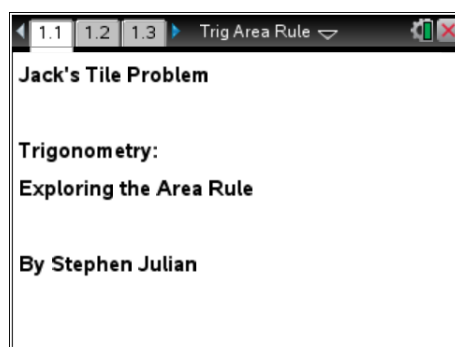
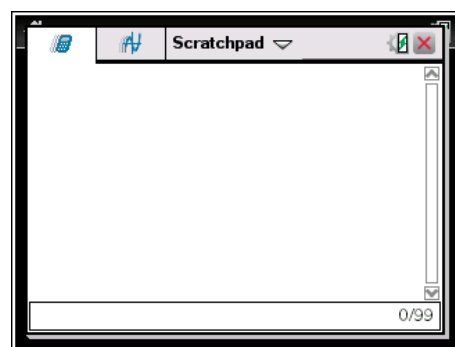
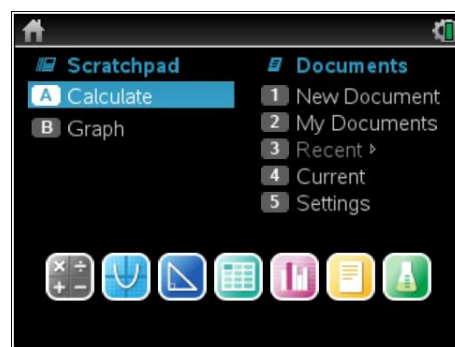
Part of in this investigation requires calculations to be performed. The Scratchpad is a place where calculations can be computed and then discarded. To access the Scratchpad press **[home]** and select **Scratchpad** (or press **[A]**). Alternatively, press the  key (this key is not available on a Clickpad).

Open the “Trig Area Rule” file. Page **1.1** is the introductory page.

Explanation of the Problem

Read pages **1.2** to **1.6** to outline the problem you are about to explore.

These pages include explanations to the student about the calculator steps they will be using.

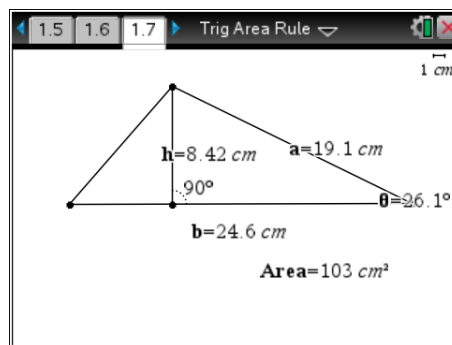


Manipulation of the Triangle and Recording of Results

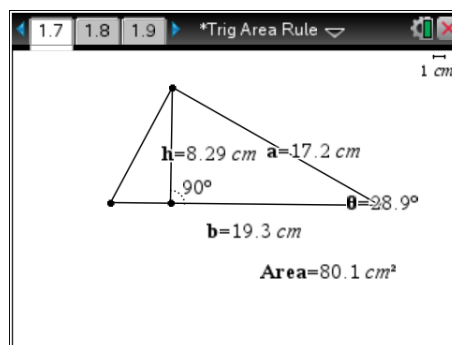
Navigate to page 1.7. Grab and move the vertices of the triangle to form different triangles.

Once you are happy with a triangle, you can record the results into the table on page 1.8 by pressing **[ctrl] + [.]**.

This action will capture the base, height and area measurements for a given triangle.



Repeat this process four more times (ensure the height measurement is always visible or it will not be recorded). You will now have captured the base, height and area measurements for five different triangles.



Viewing the Results

Navigate to page 1.8 to view the results of the data capture. If you decide to delete and then recapture a set of measurements, highlight and delete a row of data. When prompted to overwrite the data, select 'Yes'. Navigate to page 1.7 and capture any new data.

When you have finished collecting data for five different triangles, record your results in the table for Question 1.

	base	height	area1
1	24.55	8.423	103.4
2	19.32	8.295	80.14
3	14.39	7.176	51.64
4	24.08	10.79	129.9
5	25.86	12.61	163.

Each student should get different results.

Looking at the Area Rule

Read pages 1.9 and 1.10 to learn about the area rule for a triangle given two sides and the included angle.

Page 1.11 shows the measurements for the five triangles you created. However this time the area has been calculated using the formula $\text{Area} = \frac{1}{2} ab \sin(\theta)$ (delete any extra rows of measurements you may have created).

Copy these results to the table in Question 3.

Complete Questions 4, 5 and 6 below.

	sidea	sideb	angleθ	area2
1	19.13	24.55	26.13	103.4
2	17.17	19.32	28.89	80.14
3	13.15	14.39	33.07	51.64
4	18.9	24.08	34.8	129.9
5	20.93	25.86	37.04	163.

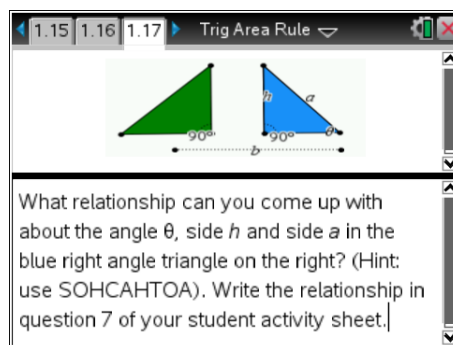
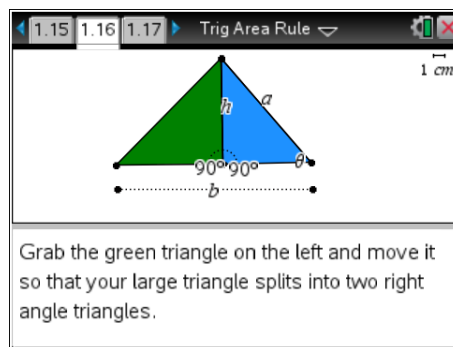
Proof of the Area Rule

Read pages 1.14 to 1.18 to see a “Proof of the Area Rule”. The triangle can be separated by grabbing the green triangle and moving it to the left.

The student should see that the given triangle is made up of two right angled triangles.

After reading these pages and moving the triangles, complete Questions 7, 8 and 9 below to see if you understand how this area rule works.

This is done by using the sine ratio combined with $\text{Area} = \frac{1}{2}bh$.



Questions

- Record your results from page 1.8 on your TI-Nspire in the table below.

Student's answers will vary depending on the generated values from their TI-Nspire CAS.

Triangle	Base	Height	Area
1	24.55	8.423	103.4
2	19.32	8.295	80.14
3	14.39	7.176	51.64
4	24.08	10.79	129.9
5	25.86	12.61	163

2. Use the base and height measurement from the table in question 1 and the formula

$$\text{Area} = \frac{1}{2} \text{ base } \times \text{ height}$$

to check the area values that were generated on your TI-Nspire CAS. Use the Scratchpad for your calculations. Clearly show your working for each triangle in the spaces provided below.

Student's answers will vary depending on the generated values from their TI-Nspire CAS. Answers should match the values in their table in question 1.

Triangle 1:

$$\text{Area} = \frac{1}{2} \times b \times h$$

$$= \frac{1}{2} \times 24.55 \times 8.432$$

$$= 103.503 \text{ cm}^2$$

Triangle 2:

$$\text{Area} = \frac{1}{2} \times b \times h$$

$$= \frac{1}{2} \times 19.32 \times 8.295$$

$$= 80.1297 \text{ cm}^2$$

Triangle 3:

$$\text{Area} = \frac{1}{2} \times b \times h$$

$$= \frac{1}{2} \times 14.39 \times 7.176$$

$$= 51.6313 \text{ cm}^2$$

Triangle 4:

$$\text{Area} = \frac{1}{2} \times b \times h$$

$$= \frac{1}{2} \times 24.08 \times 10.79$$

$$= 129.912 \text{ cm}^2$$

Triangle 5:

$$\text{Area} = \frac{1}{2} \times b \times h$$

$$= \frac{1}{2} \times 25.86 \times 12.61$$

$$= 163.047 \text{ cm}^2$$

3. Record your results from page 1.11 on your TI-Nspire CAS in the table below.

Student's answers will vary depending on the generated values from their TI-Nspire CAS. The area values in this table should match the values in question 1 and 2. This shows the student that the two methods give the same answer, however the formulas are dependent on different information from the triangles.

Triangle	Side A	Side B	Angle θ	Area
1	19.13	24.55	26.13	103.4
2	17.17	19.32	28.89	80.14
3	13.15	14.39	33.07	51.64
4	18.9	24.08	34.8	129.9
5	20.93	25.86	37.04	163

4. Use the *side a*, *side b* and *angle θ* values from the table in question 3 and the formula

$$\text{Area} = \frac{1}{2} \times a \times b \times \sin(\theta)$$

to check the area values that were generated on your TI-Nspire CAS. Use the Scratchpad for your calculations (ensure your calculator is set to **Degree** mode).

Clearly show your working for each triangle in the spaces provided below.

Student's answers will vary depending on the generated values from their TI-Nspire CAS. Answers should match the values in their table in question 3.

Triangle 1:

$$\text{Area} = \frac{1}{2} \times a \times b \times \sin(\theta)$$

$$= \frac{1}{2} \times 19.13 \times 24.55 \times \sin(26.13)$$

$$= 103.417 \text{ cm}^2$$

Triangle 2:

$$\text{Area} = \frac{1}{2} \times a \times b \times \sin(\theta)$$

$$= \frac{1}{2} \times 17.17 \times 19.32 \times \sin(28.29)$$

$$= 80.1329 \text{ cm}^2$$

Triangle 3:

$$\text{Area} = \frac{1}{2} \times a \times b \times \sin(\theta)$$

$$= \frac{1}{2} \times 13.15 \times 14.39 \times \sin(33.07)$$

$$= 51.6275 \text{ cm}^2$$

Triangle 4:

$$\text{Area} = \frac{1}{2} \times a \times b \times \sin(\theta)$$

$$= \frac{1}{2} \times 18.9 \times 24.08 \times \sin(34.8)$$

$$= 129.869 \text{ cm}^2$$

Triangle 5:

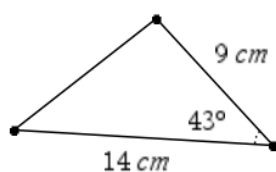
$$\text{Area} = \frac{1}{2} \times a \times b \times \sin(\theta)$$

$$= \frac{1}{2} \times 20.93 \times 25.86 \times \sin(37.04)$$

$$= 163.017 \text{ cm}^2$$

5. Using $\text{Area} = \frac{1}{2} \times a \times b \times \sin(\theta)$ calculate the area of the triangles below, showing full working out and giving your answers correct to two decimal places where necessary.

(a)

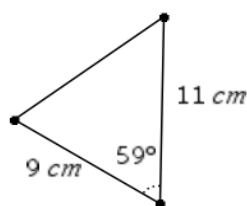


$$\text{Area} = \frac{1}{2} \times a \times b \times \sin(\theta)$$

$$= \frac{1}{2} \times 14 \times 9 \times \sin(43)$$

$$= 42.97 \text{ cm}^2$$

(b)

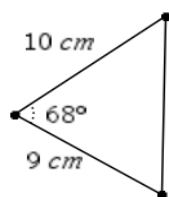


$$\text{Area} = \frac{1}{2} \times a \times b \times \sin(\theta)$$

$$= \frac{1}{2} \times 11 \times 9 \times \sin(59)$$

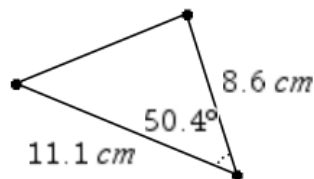
$$= 42.43 \text{ cm}^2$$

(c)



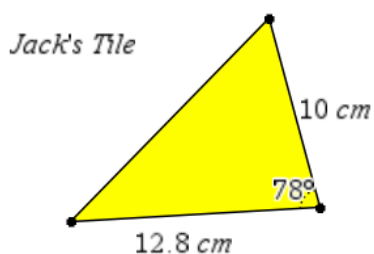
$$\begin{aligned} \text{Area} &= \frac{1}{2} \times a \times b \times \sin(\theta) \\ &= \frac{1}{2} \times 10 \times 9 \times \sin(68) \\ &= 41.72 \text{ cm}^2 \end{aligned}$$

(d)



$$\begin{aligned} \text{Area} &= \frac{1}{2} \times a \times b \times \sin(\theta) \\ &= \frac{1}{2} \times 11.1 \times 8.6 \times \sin(50.4) \\ &= 36.78 \text{ cm}^2 \end{aligned}$$

6. Help Jack find the area of his triangular tile.



$$\begin{aligned} \text{Area} &= \frac{1}{2} \times a \times b \times \sin(\theta) \\ &= \frac{1}{2} \times 12.8 \times 10 \times \sin(78) \\ &= 62.60 \text{ cm}^2 \end{aligned}$$

7. What is the relationship between the angle θ , side h and side a in the blue triangle on page 1.17 of your TI-Nspire CAS? (Hint: use SOHCAHTOA)

$$\text{The relationship is } \sin(\theta) = \frac{h}{a}$$

8. Rearrange your relationship in question 7 to make h the subject.

$$h = a \sin(\theta)$$

9. It is known that the area of a triangle can be found using $\text{Area} = \frac{1}{2} \times b \times h$.
Substitute your relationship from question 8 where h was the subject into $\text{Area} = \frac{1}{2} \times b \times h$.
What rule do you get?

$$\begin{aligned} \text{Area} &= \frac{1}{2} \times b \times h \\ &= \frac{1}{2} \times b \times (a \times \sin(\theta)) \\ &= \frac{1}{2} \times a \times b \times \sin(\theta) \end{aligned}$$