Going around in Circles

Teachers Teaching with Technology™
Professional Development from Texas Instruments

Teacher Answers

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TI-Nspire™ Investigation Student 50 min

Introduction

Memorising lots of geometric facts can be challenging, understanding where the relationships reduces the amount of information to be memorised and helps dissect new problems. In this activity you will learn how to prove some simple circle theorems. Hints are provided to help you progress through the proof.

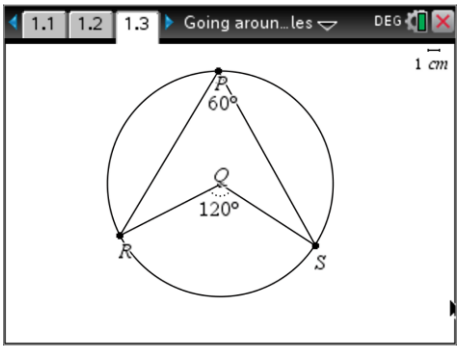
From Observation to Proof

Open the TI-Nspire document: "Going around in Circles".

Navigate to page 1.3, grab point R and move it around the circle and observe the two angles: ∠RPS and ∠RQS.

Repeat this process with points P and S and continue to observe the measured angles: ∠RPS and ∠RQS.

Note: Point Q is at the centre of the circle.

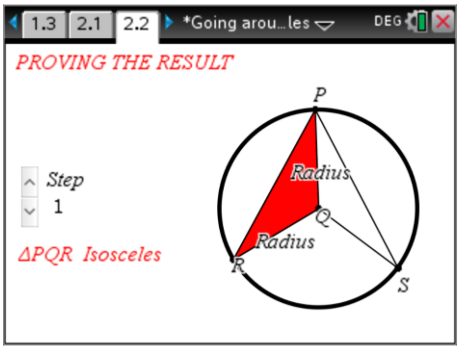


Question: 1

Write down your observed relationship between angles ∠RPS and ∠RQS.

Answer: Students should observe that ∠RPS = ∠RQS.

Navigate to page 2.2. The slider labelled "Step" can be used to work through a proof of the relationship.



The first step shows an isosceles triangle. The word 'radius' appears on two sides of the triangle as justification that the shaded triangle has two equal sides and is therefore isosceles. What does this indicate about angles: ∠QRP and ∠QPR? Check out the next step.

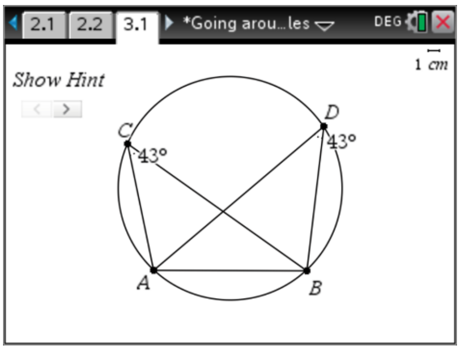
Continue stepping through the proof. It may also help if you draw some diagrams and write some notes as you work through the proof.

Navigate to page 3.1 where another circle property is displayed.

Notice that ∠ACB = ∠ADB.

Is this always true? Move points A, B, C and D around to observe the relationship.

Once you have observed the relationship, move point C to the top of the circle and point D to the side, then click on the "Hint" button.



Question: 2

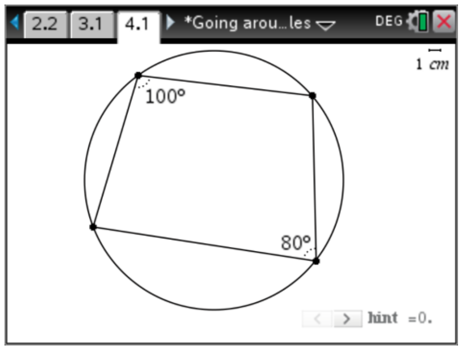
Explain how the circle relationship on page 3.1 relates to the circle relationship in problems 1 and 2.

Answer: Let Q be the centre of the circle. From the previous problem ∠AQB = 2 × ∠ACB and ∠AQB = 2 × ∠ADB therefore ∠ACB = 2 × ∠ADB.

Navigate to page 4.1 where another circle property is displayed.

Property: Opposite angles in a cyclic quadrilateral add to 180°.

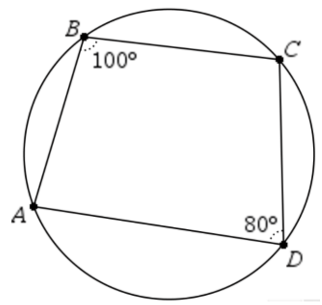
Once again the "hint" button provides a source of ideas.



Question: 3

Prove that the opposite angles in a cyclic quadrilateral are supplementary (Add to 180°).

Answer: 'Hints' on page 4.1 provide most of the required proof.



Line 1: ∠ABD = ∠ACD — Angles subtended at the circumference by the same chord are equal.

Line 2: ∠BAC = ∠BDC — Angles subtended at the circumference by the same chord are equal.

Line 3: ∠CAD = ∠CBD — Angles subtended at the circumference by the same chord are equal.

Line 4: ∠BCA = ∠BDA — Angles subtended at the circumference by the same chord are equal.

Line 5:

∠ABD + ∠DBC + ∠BCA + ∠ACD + ∠CDB + ∠BDA + ∠DAC + ∠CAB = 360°

Sum of angles in quadrilateral = 360°

Line 6: Combining Lines 1 to 5.

2 × ∠ABD + 2 × ∠DBC + 2 × ∠CDB + 2 × ∠BDA = 360°

(∠ABD + ∠DBC) + (∠CDB + ∠BDA) = 180°

∠ABC + ∠CDA = 180°

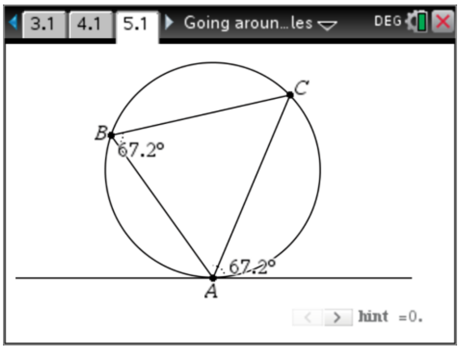
Tip

Euclidean Geometry is based on a small set of 'intuitive' axioms. Theorems are successively constructed (proved) based only on prior axioms and theorems. When you are trying to prove your observations: "opposite angles in a cyclic quadrilateral are supplementary" you can use theorems that have already been proven. Hints on page 4.1 provide suggestions on how this might be accomplished.

Navigate to page 5.1 where another circle property is displayed.

Property: "Alternate segment theorem – Angles are equal."

The "hint" button provides a source of ideas.



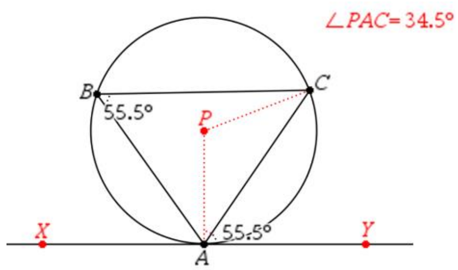
Tip

When trying to prove a circle theorem, a useful first step is to draw radii to any points on the circumference involved in the theorem. The 'alternate segment' diagram on page 5.1 shows three points on the circumference but does not show any radii.

Question: 4

Prove that alternate segment theorem.

Answer: 'Hints' on page 5.1 provides a guide for students.



Line 1: ∠YAC + ∠CAP = 90° [XY is tangent to the circle]

Line 2: ∠CAP = ∠ACP [Isosceles Triangle]

Line 3: ∠APC = 180 – 2 × ∠CAP [Sum of angles in triangle = 180°]

Line 4: ∠ABC = ½ × ∠APC [Central angle is twice the subtended angle by the same chord]

Line 5: ∠ABC = ½ × (180 – 2 × ∠CAP) [Combine lines 3 and 4]

∠ABC = 90 – ∠CAP

Line 6: ∠ABC = ∠YAC [Combining lines 1 and 5]