



Math Objectives

- Students will graph linear functions and determine properties such as slope and y-intercept, describe how both change as the graph is rotated and translated, and understand that the coordinates of a line are the solution set to the equation of the line.
- Students will show that lines with equal slopes are parallel and slopes that are negative reciprocals are perpendicular.
- Students will try to make a connection with how to understand these topics in IB Mathematics courses and on their final assessments.

Vocabulary

- Parallel
- Perpendicular
- Reciprocal
- Slope

About the Lesson

- This lesson is aligning with the curriculum of IB Mathematics Applications and Interpretations SL/HL and IB Mathematics Approaches and Analysis SL/HL
- This falls under the IB Mathematics Core Content Topic 2 Functions:
 - 2.1:** (a) Different forms of the equation of a straight line
 - (b) Gradient; intercepts
 - (c) Lines with gradients m_1 and m_2
 - (d) Parallel lines $m_1 = m_2$
 - (e) Perpendicular lines $m_1 \times m_2 = -1$

As a result, students will:

- Apply this information to real world situations.

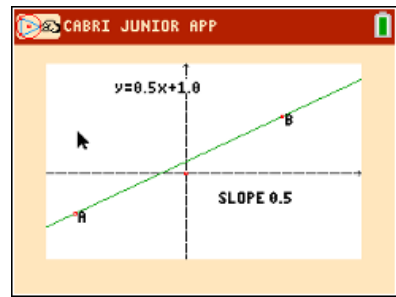
Teacher Preparation and Notes.

- This activity is done with the use of the TI-84 family as an aid to the problems and Cabri Jr. will be used throughout.

Activity Materials

- Compatible TI Technologies: TI-84 Plus*, TI-84 Plus Silver Edition*, TI-84 Plus C Silver Edition, TI-84 Plus CE

* with the latest operating system (2.55MP) featuring MathPrint™ functionality.



Tech Tips:

- This activity includes screen captures taken from the TI-84 Plus CE. It is also appropriate for use with the rest of the TI-84 Plus family. Slight variations to these directions may be required if using other calculator models.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at <http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials>

Lesson Files:

Student Activity
 PointsLinesSlopesOhMy_Stude
 nt-84CE.pdf
 PointsLinesSlopesOhMy_Stude
 nt-84CE.doc
 PARALLEL.8xp
 PERPENDI.8xp



Tech Tip: Before beginning the activity, the files PARALLEL.8xv and PERPENDI.8xv need to be transferred to the students' calculators via handheld-to-handheld transfer or transferred from the computer to the calculator via TI-Connect.

One way to move from point A to point B is by first going in a vertical direction and then horizontally. When points A and B are on the same line, a special relationship exists between the vertical and horizontal moves. In this activity, you will use coordinates to better understand that relationship, as well as the relationship between coordinates of points and their quadrant locations, slopes and y-intercepts, and parallel and perpendicular lines.

Problem 1 – Coordinates of Points

Open the *Cabri™ Jr* application by pressing **apps**. Open a new file (**y =, New**) and make sure the axes are displayed (press **graph, Hide/Show, Axes**). Note, to undo press **F1, Undo**.

Place a point, *P*, in the top right quadrant. Use **F2, Point, Point On**. Press **F5, Alpha-Num** and **enter** to label the points. When finished with a tool press **clear**.

Display the coordinates of the points. Use **F5, Coord & Eq** and move the cursor to a point until the point is flashing. Press **enter** to select that point, then move the cursor to where you want the coordinate to remain and press **enter** again.

To grab a point that is flashing, press **alpha**. To let go of a point press **clear**. This works as Escape.

First, a little review on coordinate location within the four quadrants. Drag point *P* around into different quadrants. Complete the sentences by writing *positive* or *negative*.

1. A point is in Quadrant 1 (top right) when its x-coordinate is _____ and its y-coordinate is _____. **Solution:** positive and positive
2. A point is in Quadrant 2 (top left) when its x-coordinate is _____ and its y-coordinate is _____. **Solution:** negative and positive
3. A point is in Quadrant 3 (bottom left) when its x-coordinate is _____ and its y-coordinate is _____. **Solution:** negative and negative
4. A point is in Quadrant 4 (bottom right) when its x-coordinate is _____ and its y-coordinate is _____. **Solution:** positive and negative

Teacher Tip: Students should now be able to make sense of exactly what the coordinates should be in different quadrants as well as on each axis. Give students the coordinates of a



point and ask which quadrant it is in and visa versa. This simple activity is good for students to develop their technology skills and establish sign patterns for the four quadrants.

Problem 2 – Slope Review

Place a second point, Q , on your graph in Quadrant 1. Show its coordinates as you did with point P in problem 1. Move P into Quadrant 3. Use **F2, Line** to draw a line connecting P and Q .

Suppose you wanted to go from P to Q , but you could only make a vertical move up or down or a horizontal move right or left.

5. Describe the vertical and horizontal moves you would make to get from point P to point Q .

Solution: Answers will vary, but you would move up and to the right.

6. a. Move point P until the vertical path to a horizontal line through Q is up approximately 2 spaces. Describe the horizontal move to get to point Q .

Solution: Answers will vary, but students should say how many units to the right it would take.

- b. Move point P until the vertical path to a horizontal line through Q is up approximately 3 spaces. Describe the horizontal move to get to point Q .

Solution: Answers will vary, but students should say how many units to the right it would take.

- c. Write down the horizontal move that will correspond to a vertical move of up approximately 6. Move the point to check your answer.

Solution: Answers will vary, but students should say how many units to the right.

Teacher Tip: Students should realize that when moving from one point to another, the vertical and horizontal moves involve direction as well as distance. Be sure students remember that the quadrants are labeled from 1 to 4 counterclockwise – beginning with the first quadrant where x and y are both positive. And be sure that the students are describing both the distance and the direction for this problem.

7. Make an educated guess about the relationship between the number of units and the direction from P to Q . Choose some new points for P and Q , and verify your conjecture.



Possible Answers: The number of units vertically and horizontally from P to Q will vary depending on the points created by the students in the app. Students might note that if the direction from P to Q is up, then the line has a positive slope. If the direction from P to Q is down, then the line has a negative slope.

8. Describe what happens when point P is above and to the right of point Q . Try several points for which this is true. Explain if the results support your conjecture.

Possible Answers: The vertical change is down, and the horizontal change is to the left. The student could count and observe the direction from point to point.

In a coordinate system, a move up is considered a positive vertical change; a move down is a negative vertical change; a move right is considered a positive horizontal change; a move left is a negative horizontal change.

9. Using correct signs, find the ratio of vertical change to horizontal change for several pairs of points on the line. Explain what you observe about the ratios.

Possible Answer: The ratios are equivalent.

10. Move points P and Q to fill in the missing information in each line of the table below. Explain your reasoning.

	Coordinates of Point P	Coordinates of Point Q	$\frac{\text{Vertical Change}}{\text{Horizontal Change}}$
1	(-8 , -2)	(6 , 5)	$\frac{7}{14}$
2	(-6 , -1)	(-2 , 1)	$\frac{2}{4}$
3	(2 , 3)	(8 , 6)	$\frac{3}{6}$
4	(6 , 5)	(-6 , -1)	$\frac{-6}{-12}$

Teacher Tip: Students might struggle with the fact that a ratio with two negative signs is the same as the ratio where both numerator and denominator are positive.



Some of them might also be subtracting the values to find the vertical and horizontal changes, but depending on the order in which they subtract, they might get both positive or negative values. Be sure to stress that if they do this, they have to start from the same point to calculate both the horizontal and vertical change.

11. Describe how the information in the table in question 10 relates to your observations in question 9.

Solution: The ratios are equivalent.

12. Suppose points P and Q are on the line but not displayed in the window of the document. If the vertical change from point P to point Q is 50, find the horizontal change. Explain your reasoning.

Possible Solution: The horizontal change would be 100. The ratios are equivalent so the numerator is 50 and the denominator has to be 100.

Teacher Tip: The vertical change of 50 means “up 50” or “positive 50.” Ask the students to consider if the vertical change is “negative 50” or if the horizontal change is 50.

13. For a different line, the coordinates of point P are $(-3, -4)$, and the ratio of the vertical change to the horizontal change is equivalent to $\frac{2}{3}$. Find the coordinates of another point on the line. Explain how you found your answer.

Possible Solutions: Many different responses are possible. $(0, -2)$, $(3, 0)$, $(6, 2)$. “I moved up 2 and right 3.”

Teacher Tip: Students should be able to show how their points are generated using the ratio $2/3$. Sharing responses will help them realize that the ratio of the vertical change to the horizontal change for any two points on a line is constant.

Problem 3 – Lines, Equations, and Slopes



Using tools from the F5 find the equation and slope of the line PQ . Press **plus** as you hover over a value to display more digits.

Look for relationships between the points, slope, and equation as you change the line by grabbing and dragging point P , and then by grabbing and dragging the line itself.

14. Place a point on the line and label its coordinates under **F5**. Drag the point along the line and record several coordinates of points. Explain how the coordinates relate to the equation of the line.

Solution: Coordinates will vary. Each ordered pair is a solution to the equation of the line.

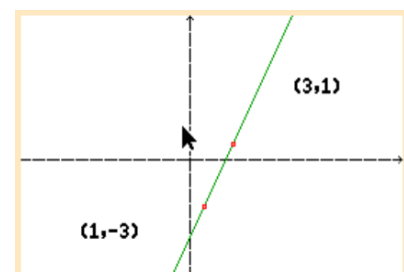
15. When dragging the line by point P , describe the relationship between the points and the slope.

Solution: The slope is the change in the y -values divided by the change in the x -values.

16. When dragging the line by a point, describe the relationship of the slope and the equation.

Solution: The slope changes. The coefficient in front of the variable x changes.

17. To the right is a graph with two points labeled. Consider the line through these points. Then, consider the graph of the equation $y = \frac{1}{2}x + 2$. Show your work and explain how the two lines compare. Especially consider the slope and y -intercepts.



Solution: The slope of the graph is $(1 - (-3))/(3 - 1) = 4/2 = 2$. This is a larger slope than the equation which has a slope of $\frac{1}{2}$. The graph is much steeper than the equation. The y -intercept of the equation is positive 2, whereas the y -intercept of the graph is a negative number. From the equation of the graph, $y + 3 = 2(x - 1)$ or $y = 2x - 5$, it can be seen the y -intercept is -5 .

Problem 4 – Slopes of Parallel and Perpendicular Lines



Open the Cabri™ Jr. file **PARALLEL**. Drag the lines by points P and Q and examine the slopes.

18. Explain what you notice about the slopes of two parallel lines.

Solution: The slopes of the two parallel lines are equal.

Open the Cabri™ Jr. file **PERPENDI**. Drag the lines to investigate the relationship between the slopes.

19. Explain what you notice about the slopes of two perpendicular lines.

Solution: Students will observe that when one slope is positive, the other slope is negative. They may need to continue exploring to discover that the slopes of the perpendicular lines are the opposite reciprocals of each other.

Discuss with a classmate what happens when the slopes of two perpendicular lines are multiplied together. Test your theory on the current slopes of both lines. Now, change the lines by grabbing and dragging point P and test your theory several times.

20. Describe what you observe about the product of the slopes.

Solution: The product is always -1 . The slope of perpendicular lines is the opposite reciprocal of each other.

Further IB Application

Hannah has always liked the kite shape. She plans to tile her bathroom floor with a pattern made up of kites. For the patten, she will be designing her initial kite $PQRS$ on a set of coordinate axes in which one unit represents 5 cm.

The coordinates of P , Q , and R are $(2, 0)$, $(0, 4)$, and $(4, 6)$ respectively. Point S lies on the x -axis. PR is perpendicular to QS . See the diagram below.

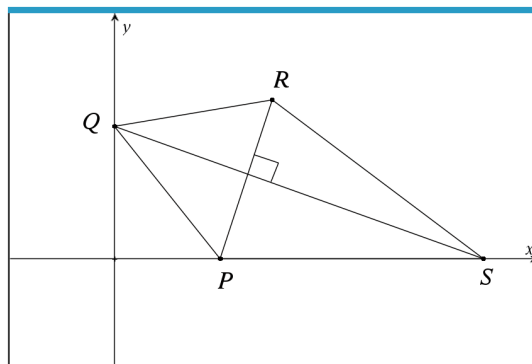


Diagram not to scale.

(a) Find the gradient of PR .



Solution: $m = \frac{6-0}{4-2} = 3$

(b) Write down the gradient of QS.

Solution: $m = -\frac{1}{3}$

(c) Find the equation of QS and write the answer in the following forms:

(i) $y = mx + c$

Solution: $y = -\frac{1}{3}x + 4$

(ii) $y - y_1 = m(x - x_1)$

Solution: $y - 4 = -\frac{1}{3}(x - 0)$

(iii) $ax + by + d = 0$

Solution: $x + 3y - 12 = 0$

(d) Find the coordinates of point S.

Solution: $(12, 0)$

Teacher Tip: Please know that in this activity there is a lot of time dedicated to students talking with one another and sharing their thoughts with the class. The goal here is to not only review linear functions and their characteristics, but also to generate discussion.

***Note: This activity has been developed independently by Texas Instruments and aligned with the IB Mathematics curriculum, but is not endorsed by IB™. IB is a registered trademark owned by the International Baccalaureate Organization.*