Math Objectives

- Students will relate the inequality symbol to the location of points on a number line.
- Students will recognize, moreover, that the value of a number depends on its position on the number line and that numbers farther to the left are smaller.
- Students will recognize what operations on an inequality reverse the direction of the inequality.
- Students will use appropriate tools strategically (CCSS Mathematical Practices).
- Students will look for and express regularity in repeated reasoning (CCSS Mathematical Practices).

Vocabulary

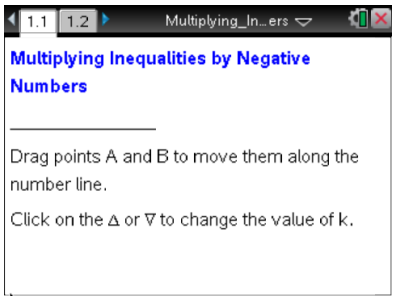
- inequality statement

About the Lesson

- In this lesson students investigate the effect of multiplying the numbers on a number line by a negative number and the symbolic and graphical representations of the solutions of an inequality.
- Students will change the value of a multiplier (*k*) and observe the effects on the position of the resulting number on the number line and in the inequality statements. As a result, students will recognize what operations on an inequality reverse the direction of the inequality.

TI-Nspire™ Navigator™ System

- Use Quick Poll to check student understanding.
- Use Screen Capture to examine patterns that emerge.
- Use Live Presenter to engage and focus students.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Tech Tips:

- Make sure the font size on your TI-Nspire handhelds is set to Medium.
- You can hide the function entry line by pressing ctrl G.

Lesson Materials:

Student Activity

Multiplying_Inequalities_by_Negative_Numbers_Student.pdf

Multiplying_Inequalities_by_Negative_Numbers_Teacher.doc

TI-Nspire document

Multiplying_Inequalities_by_Negative_Numbers_.tns

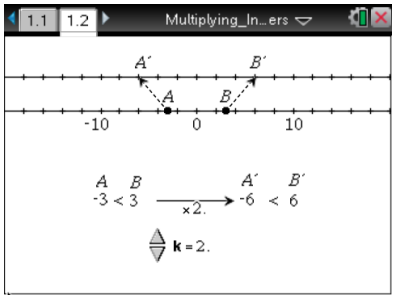
Visit www.mathnspired.com for lesson updates and tech tip videos.

Discussion Points and Possible Answers

Tech Tip: If students experience difficulty dragging a point, check to make sure that they have moved the cursor arrow close to the point. The arrow should become a hand ( ) getting ready to grab the point. Instruct students to press ctrl to grab the point and close the hand. After the point has been moved, they should press esc to release the point.

Note: On the number line, the movement of points *A* and *B* is restricted to integral values between –5 and 5, inclusive. The multiplier (*k*) is restricted to integral values between –4 and 4, inclusive, and is adjusted by clicking on the up or down symbol (Δ or ∇).

Move to page 1.2.



1. a. How is the inequality between *A* and *B* related to the inequality between *A′* and *B′*
Answer: The numbers in the inequality on the right are twice those in the inequality on the left.

   b. How is *A′* related to *A* on the number lines?
Answer: Possible answer: *A′* is *A* multiplied by 2.

2. By clicking on the up or down symbol on the screen (Δ or∇), change the multiplier *k* to 3.
   a. What is the relationship between *A* and *B* related to the inequality between *A′* and *B′*?
Answer: The inequality on the right is 3 times as large as the inequality on the left.

   b. How is *A* related to *A*' and *B* to *B*'?
Answer: Sample Answer: *A′* is –9 while *A* is –3; it is three times farther from 0 than *A*. The same thing is true for *B* and *B*'.

Teacher Tip: Ask students to consider how the distance A and *A'* are from 0 changed when you increased *k* from 2 to 3. It is important that they recognize that the distance from 0 determines the magnitude of a number and that the size or value of the number is determined by how far it is to the left on the number line (i.e., the farther to the left, the smaller the value, explaining why –5 is less than –1 even though it has a larger magnitude).

3. a. Move point *A* from –5 to 5, and describe the changes that occur.

Answer: *A*' moves at the same time and its distance from *A* changes. The first number in the inequality changes. When *A* is to the left of *B*, the inequality symbol is "<." When *A* is to the right of *B*, the inequality symbol is ">." If *A* is moved to the same position as *B*, *A* and *A*' seem to disappear and the inequality symbol becomes an equals sign.

b. Move point *B* to 5, and describe the changes that occur.

Answer: *B*' moves at the same time, and its distance from *B* changes. The second number in the inequality changes. When *A* is to the left of *B*, the inequality symbol is "<." When *A* is to the right of *B*, the inequality symbol is ">." If *B* is moved to the same position as *A*, the inequality symbol becomes an equals sign.

Teacher Tip: If *A* is moved to the same position as *B*, *A,* and *A*' seem to disappear, both points are still there. Actually *B* and *B*' are layered on top of *A* and *A*'. *A* can still be moved.

4. Vary the value of the multiplier *k* between –4 and 4. Identify all values for *k* in the following.

a. $A' = B'$

Answer: $k = 0$

b. $A' = A$, $B' = B$

Answer: $k = 1$

c. A' and B' are in the same order as *A* and *B*

Answer: $k > 0$

d. A' and B' are in a different order than *A* and *B*

Answer: $k < 0$

Teacher Tip: Encourage students to look both at the arrows and the inequalities and observe what is happening. Students may benefit from a discussion of why a multiplier of 0 makes the arrows converge and why a multiplier of 1 causes the arrows to be vertical.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 1 at the end of this lesson.

5. Sara multiplied both sides of the inequality $1 < 5$ by –1. Her answer was $–1 < –5$. Is she correct? Explain.

   Answer: Sara is not correct. If you multiply each number of the inequality by –1, the results is $–1 > –5$. The reason is that the magnitude remains constant but the position of the numbers on the number line are reversed. So, therefore, is the direction of the inequality.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 2 at the end of this lesson.

Teacher Tip: This is a critical discussion to have related to the tip in question 1b about the difference between the magnitude and the value of a number.

6. Complete the table below. For each description, state the inequality in column 2. Then describe how each value of *k* affects the relation on the right..The first row is partly done for you.

Description	Inequality	$k = 3$	$k = 0$	$k = 2$	$k = –4$
Point *A* is left of point *B*	$1 < 5$ where A is 1 and *B* is 5	$3 < 15$	**0 = 0**	**2 < 10**	**–4 > –20**
Point *A* is right of point *B* and point *B* is less than 0	**4 > –1 where *A* is 4 and *B* is –1**	**12 > –3**	**0 = 0**	**8 > –2**	**–16 < 4**
Point *A* is negative and point *B* is positive	**–2 < 3 where *A* is –2 and *B* is 3**	**–6 < 9**	**0 = 0**	**–4 < 6**	**8 > –12**
Both points *A* and *B* are negative	**–2 > –5 *A* is –2 and *B* is –5**	**–6 > –15**	**0 = 0**	**–4 > –10**	**8 < 20**

TI-Nspire Navigator Opportunity: *Live Presenter*
See Note 3 at the end of this lesson.

7. In general, what value(s) of *k* will result in:
   a. The relationship in both inequalities being the same?
      Answer: k > 0

   b. The inequality symbol on the left becoming an equals sign on the right?
      Answer: k = 0

   c. The relationship in the inequalities being reversed?
      Answer: k < 0

8. Write an inequality comparing the two given numbers. Then complete the table using what you have learned about working with inequalities.

Numbers	Inequality	Multiply both sides by –3	Multiply both sides by 4	Add 2 to both sides	Subtract 3 from both sides	Divide both sides by –1
–1 , –10	**–10 < –1**	**30 > 3**	**–40 < –4**	**–8 < 1**	**–13 < –4**	**10 > 1**
2 , –10	**–10 < 2**	**30 > –6**	**–40 < 8**	**–8 < 4**	**–13 < –1**	**10 > –2**
–4 , –8	**–8 < –4**	**24 > 12**	**–32 < –16**	**–6 < –2**	**–11 < –7**	**8 > 4**

Teacher Tip: Be sure students recognize that multiplying and dividing by a negative number will reverse the inequality sign, but adding and subtracting will not.

9. Tell whether the statement is always true, sometimes true, or never true. Give a reason for your thinking. (It might help to choose values for *a*, *b,* and *c* and test them with or without the TI-Nspire document file.)
   a. If $a < b$, then $ac > bc$.
      Answer: Sometimes true. The statement is only true if *c* is less than 0.

   b. If $a \geq b$, then $b \leq a$.
      Answer: Always true. The statement is always true because the statements "*a* is greater than or equal to *b*" and "*b* is less than or equal to *a*" have the same meaning.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students are able to understand that:

- Given any set of positive or negative numbers, the one further to the left on the number line is the smallest.
- The product of two negative numbers becomes larger than either of the original numbers.
- When both sides of an inequality are multiplied by a negative value, the direction of the inequality is reversed.

TI-Nspire Navigator

Note 1

Question 4, *Quick Poll*: Use the Open Response feature of Quick Poll and have students submit their answers to questions 4a, 4b, 4c, 4d (one at a time) and discuss the answers.

Note 2

Question 5, *Quick Poll*: Use the Yes No feature of Quick Poll and have students submit their answer to question 5. Discuss why the answer is no.

Note 3

Question 6, *Live Presenter*: If students need to have any of the parts of number 6 clarified, assign a student to be the Live Presenter and illustrate his/her answers