Math Objectives

- Students will use their knowledge of matrices and their inverses to encode and decode messages, solve systems of equations, and write equations for messages.
- Students will be able to input their data into matrices and perform calculations on them.
- Students be able to interpret the results of their calculations.
- Students will utilize different methods to check the results of their calculations and verify their solutions.
- Students will apply the mathematics they know to solve problems arising in everyday life (CCSS Mathematical Practice).
- Students will create a coherent representation for a problem (CCSS Mathematical Practice).
- Students will use technological tools to explore and deepen understanding of concepts (CCSS Mathematical Practice).

Vocabulary

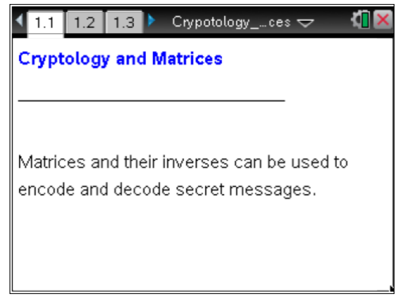
- decode
- determinant
- encode
- identity matrix
- inverse matrix
- matrix

About the Lesson

- This lesson involves using matrices to encode and decode a message.
- As a result, students will:
  - Learn how matrices can be applied to real world problems.
  - Learn that (some) matrix multiplications can be "undone" by multiplying by an inverse matrix.

TI-Nspire™ Navigator™ System

- Transfer a File.
- Use Live Presenter to provide assistance to students throughout the activity.
- Use Screen Capture to monitor students' progress.
- Use Quick Poll to assess students' understanding.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Tech Tips:

- Make sure the font size on your TI-Nspire handhelds is set to Medium.
- You can hide the function entry line by pressing ctrl G.

Lesson Files:

Student Activity

Cryptology_and_Matrices_Student.pdf

Cryptology_and_Matrices_Student.doc

TI-Nspire document

Cryptology_and_Matrices.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.

Discussion Points and Possible Answers

Teacher Tip: Prior to beginning this activity, review with students how to organize information into matrices. This activity does not teach students the process used to set up and/or multiply matrices; rather it explains how to find the inverse matrix of an invertible 2 × 2 matrix and provides several examples of problems that can be solved using inverse matrices with TI-Nspire technology. Students should already know how to multiply matrices by hand.

Cryptology is the science of developing secret codes and using those codes for encrypting and decrypting data.[1] In June of 1929, an article written by Lester S. Hill appeared in the American Mathematical Monthly. This was the first article that linked the fields of algebra and cryptology.[2]

Today, governments use sophisticated methods of coding and decoding messages. One type of code, which is extremely difficult to break, makes use of a large matrix to encode a message. The receiver of the message decodes it using the inverse of the matrix. This first matrix is called the encoding matrix and its inverse is called the decoding matrix.[3]

In this activity, we will use a simple method for encoding a message by first assigning a numeral to each letter of the alphabet. We will represent the letter A with the numeral 1 and continue to the letter Z which will be assigned the numeral 26. We will also assign the numeral 0 to a space in the message.

[1] http://www.answers.com/topic/cryptology

[2] http://www.glassblower.info/cryptosystems-journal/HILL29.HTM

[3] http://aix1.uottawa.ca/~jkhoury/cryptography.htm

For example, using the chart to the right, the word

SYSTEM

can be written using numerals as

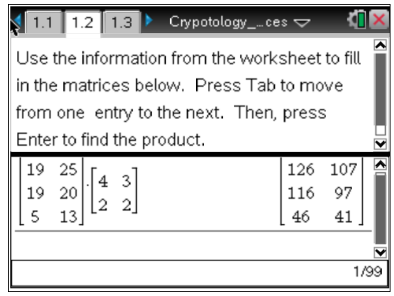
19 25 19 20 5 13

and then recorded in a matrix as

$$\begin{bmatrix} 19 & 25 \\ 19 & 20 \\ 5 & 13 \end{bmatrix}.$$

_ = 0	**I** = 9	**R** = 18
A = 1	**J** = 10	**S** = 19
B = 2	**K** = 11	**T** = 20
C = 3	**L** = 12	**U** = 21
D = 4	**M** = 13	**V** = 22
E = 5	**N** = 14	**W** = 23
F = 6	**O** = 15	**X** = 24
G = 7	**P** = 16	**Y** = 25
H = 8	**Q** = 17	**Z** = 26

Move to page 1.2.



1. To protect this message as it is transmitted, it is *encoded* by multiplying the message matrix by an encoding matrix, such as $\begin{bmatrix} 4 & 3 \\ 2 & 2 \end{bmatrix}$.

   a. Enter the following information into the matrices on Page 1.2. Press tab to move from one entry to the next.

   $$\begin{bmatrix} 19 & 25 \\ 19 & 20 \\ 5 & 13 \end{bmatrix} \cdot \begin{bmatrix} 4 & 3 \\ 2 & 2 \end{bmatrix}$$

TI-Nspire Navigator Opportunity: ***Live Presenter and Screen Capture***
See Note 1 at the end of this lesson.

   b. Press enter to find the product and fill in the spaces provided below.

   <u>**Answer:**</u> $\begin{bmatrix} 19 & 25 \\ 19 & 20 \\ 5 & 13 \end{bmatrix} \cdot \begin{bmatrix} 4 & 3 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 126 & 107 \\ 116 & 97 \\ 46 & 41 \end{bmatrix}$

   c. Fill in the numerals for the new message:

   <u>**Answer:**</u>

   _126_ _107_ _116_ _97__ __46_ __41_.

The receiver of this message can retrieve the original message by *decoding* it by using the *inverse* of the coding matrix.

The **inverse** of a 2 X 2 matrix $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, provided $\det(A)\neq 0$, is $A^{-1}=\frac{1}{\det(A)}\cdot\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

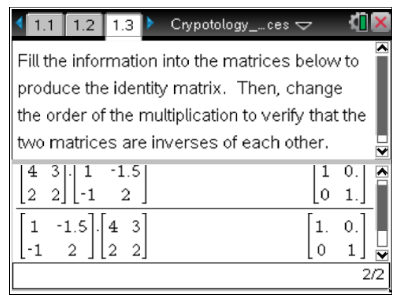
2. What is the decoding matrix (the inverse of the encoding matrix)?

   (Recall that $\det\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right)=ad-bc$.)

   Fill in the information below.

   Answer: $\begin{bmatrix} 4 & 3 \\ 2 & 2 \end{bmatrix}^{-1}=\frac{1}{\boxed{2}}\cdot\begin{bmatrix} \boxed{2} & \boxed{-3} \\ \boxed{-2} & \boxed{4} \end{bmatrix}=\begin{bmatrix} \boxed{1} & \boxed{-1.5} \\ \boxed{-1} & \boxed{2} \end{bmatrix}$.

Move to page 1.3.



3. Test the decoding matrix to verify that these matrices are inverses. (When you multiply a matrix by its inverse, in either direction, their product will be the identity matrix.) Type the appropriate entries into the matrix given on Page 1.3. Change the order of the multiplication to verify that the two matrices are inverses, and record both of the results below.

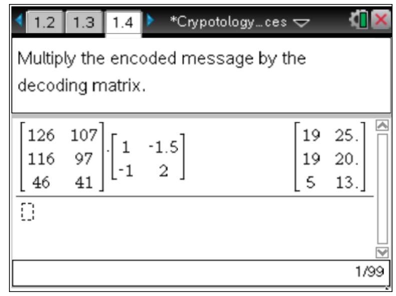
   Answer:

$$\begin{bmatrix} \boxed{4} & \boxed{3} \\ \boxed{2} & \boxed{2} \end{bmatrix}\cdot\begin{bmatrix} \boxed{1} & \boxed{-1.5} \\ \boxed{-1} & \boxed{2} \end{bmatrix}=\begin{bmatrix} \boxed{1} & \boxed{0} \\ \boxed{0} & \boxed{1} \end{bmatrix}$$

$$\begin{bmatrix} \boxed{1} & \boxed{-1.5} \\ \boxed{-1} & \boxed{2} \end{bmatrix}\cdot\begin{bmatrix} \boxed{4} & \boxed{3} \\ \boxed{2} & \boxed{2} \end{bmatrix}=\begin{bmatrix} \boxed{1} & \boxed{0} \\ \boxed{0} & \boxed{1} \end{bmatrix}$$

Teacher Tip: Multiplying the encoding matrix by the decoding matrix should result in the identity matrix, since they are inverses. Be sure that students understand that this is true regardless of the order in which the inverse matrices are multiplied.

Move to page 1.4.

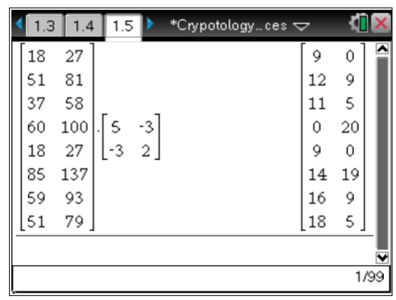


4. a. Multiply the encoded message by the decoding matrix. Record the results below.

<u>Answer:</u> $\begin{bmatrix} \boxed{126} & \boxed{107} \\ \boxed{116} & \boxed{97} \\ \boxed{46} & \boxed{41} \end{bmatrix} \cdot \begin{bmatrix} \boxed{1} & \boxed{-1.5} \\ \boxed{-1} & \boxed{2} \end{bmatrix} = \begin{bmatrix} \boxed{19} & \boxed{25} \\ \boxed{19} & \boxed{20} \\ \boxed{5} & \boxed{13} \end{bmatrix}$

b. Use the chart given at the beginning of the worksheet to see if the decoded message is the same as the original one.

<u>Sample Answer:</u> The decoded message is the word, SYSTEM, which was the original message.



5. Decode the message **18 27 51 81 37 58 60 100 18 27 85 137 59 93 51 79**, which was encoded with encoding matrix $\begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix}$.

- For this example, you will enter your own matrices into your TI-Nspire. The first matrix will have 8 rows and 2 columns. The second matrix will have 2 rows and 2 columns.
- Press `ctrl` `doc▾` **> Add Calculator**.
- Select **MENU > Matrix & Vector > Create > Matrix**, and choose a matrix with 8 rows and 2 columns. Press OK or `enter`.
- Press the right arrow (▶) twice to move outside the matrix.
- Insert a multiplication symbol to the right of the 8 X 2 matrix.
- Repeat the process for entering a matrix and choose a matrix with 2 rows and 2 columns.
- Enter the data for the encoded message into the first matrix and the data for the decoding matrix into the second.
- Press `tab` to move from one entry to the next. (Be sure that you are multiplying the given message by the decoding matrix, which is the inverse of the encoding message.)
- Press `enter` to find the product.

a. What is the decoding matrix (the inverse of the encoding matrix)?

Answer: $\begin{bmatrix} \boxed{5} & \boxed{-3} \\ \boxed{-3} & \boxed{2} \end{bmatrix}$

b. What is the matrix that has been decoded?

Answer: $\begin{bmatrix} \boxed{18} & \boxed{27} \\ \boxed{51} & \boxed{81} \\ \boxed{37} & \boxed{58} \\ \boxed{60} & \boxed{100} \\ \boxed{18} & \boxed{27} \\ \boxed{85} & \boxed{137} \\ \boxed{59} & \boxed{93} \\ \boxed{51} & \boxed{79} \end{bmatrix}$

TI-Nspire Navigator Opportunity: ***Screen Capture***   **See Note 2 at the end of this lesson.**

c. To help you decode the message, write out the numerals from the decoded message.

Answer:

__9__ __0__ __12_ __9__ __11_ __5__ __0__ __20_ __9__ __0__ __14_

__19_ __16_ __9__ __18_ __5__

d. What does the message say? (Be sure to use the chart at the beginning of this activity to decipher the results.)

Answer:

The message says:

I LIKE TI NSPIRE

TI-Nspire Navigator Opportunity: ***Quick Poll***   **See Note 3 at the end of this lesson.**

Wrap Up

Upon completion of the lesson, the teacher should ensure that students are able to understand:

- How to use matrices to encode and decode a text message.
- How to create and use their own matrices to solve problems.

Enhancement

You might want to show your students how to solve systems of simultaneous equations by using inverse matrices. The TI-Nspire activity, *Parabolas and Matrices*, focuses on this topic and also provides students with examples of how to use an inverse matrix to write an equation for a parabola when given three points that lie on its graph.

Assessment

You might want to have your students create their own algorithm to assign numerals to the letters of the alphabet and/or create their own encoding matrix. Students can work in groups to encode and decode each other's messages.

TI-Nspire Navigator

Note 1

Name of Feature: Live Presenter and Screen Capture

You might want to ask one of the students to serve as the Live Presenter and demonstrate to the class how to navigate through the activity.

You may want to leave Screen Capture running in the background, with a 30 second automatic refresh, and without student names displayed, to enable you to monitor students' progress and make the necessary adjustments to your lesson.

Note 2

Name of Feature: Screen Capture

You might want to take a Screen Capture of students' Page 1.5, so they can compare their work with the work of their peers. If many students are having difficulty, you might want to stop and better explain the process they should be following.

Note 3

Name of Feature: Quick Poll

You might want to send a Quick Poll to have students put in their decoded message(s).