Math Objectives

- Students will identify equivalent equations.
- Students will solve a system of linear equations in two unknowns by adding equivalent equations to eliminate one variable.
- Students will look for and express regularity in repeated reasoning (CCSS Mathematical Practice).

Vocabulary

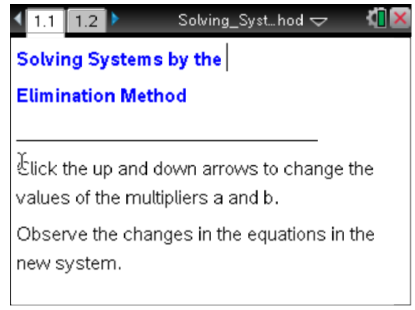
- equivalent equations
- system of linear equations
- solution to a system

About the Lesson

- This lesson involves solving a system of linear equations in two variables. The emphasis is on helping students understand how to use equivalent equations and the method of elimination to solve a system.
- Students will use slider arrows to change multipliers for a system of equations. They will use linear combinations in an effort to produce a zero sum coefficient for one of the variables and solve the resulting equation for the other. Students will find values for *x* and *y* that satisfy the original equations.
- Students can combine elimination and substitution to solve systems more efficiently. Students can determine which method is most appropriate for solving a system.

TI-Nspire™ Navigator™ System

- Use of Screen Capture/Quick Poll will allow the teacher to assess student understanding during the lesson.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Click slider arrows to change values.

Tech Tips:

- Make sure the font size on your TI-Nspire handheld is set to Medium.
- You can hide the function entry line by pressing ctrl G.

Lesson Materials:

Student Activity

- Solving_Systems_by_the_Elimination_Method_Student.pdf
- Solving_Systems_by_the_Elimination_Method_Student.doc

TI-Nspire document

- Solving_Systems_by_the_Elimination_Method_.tns

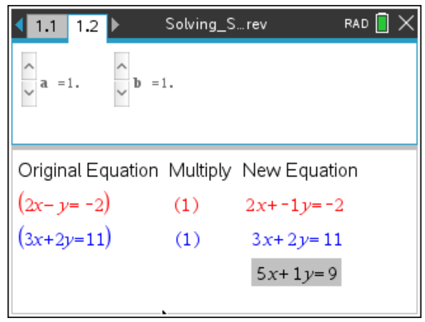
Visit www.mathnspired.com for lesson updates and tech tip videos.

Discussion Points and Possible Answers

TI-Nspire Navigator Opportunity

Let one of the students be assigned as Live Presenter to demonstrate how the minimized slider works. Take a Screen Capture of students' handhelds and refresh the screens frequently to monitor students understanding of the mathematics and to monitor their ability to use the .tns file. As you browse, look for students having difficulty and help them accordingly. It can also be helpful to pair students up, assigning students of different abilities to the same group.

Move to page 1.2.



1. Use the up or down symbols on the screen (Δ or ∇) to change the values of *a* and *b*. How do the values of *a* and *b* determine the new system?

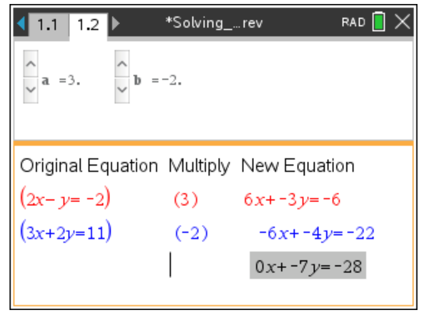
   Answers: The multipliers control the values of the multipliers *a* and *b,* respectively. As the value of *a* changes, the coefficients in the first equation of the new system, as well as the constant, change by multiples of *a*. As the value of *b* changes, the coefficients in the second equation, as well as the constant, change by multiples of *b*.

> **Teacher Tip:** Some students might simply look at the changes and then move on without thinking about the mathematics on the screen. Remind them that multiplying the whole equation by a number other than zero is the same as multiplying both sides by the same number. You might connect this to their prior experience, as they have done this before when solving equations. It is important that they realize that non-zero scalar multiplication produces an equivalent equation.

2. How is the equation in the box related to the equations above it?

   Answer: The two equations are added to get the boxed equation.

3. Change the multipliers *a* and *b* until the coefficient of *x* in the boxed equation is zero. Record your multipliers:
   a =_______, *b* = _________.



Answer: Answers will vary: example *a* = 3 and *b* = –2.

a. What must be true about the coefficients of *x* in the *new* system for their sum to be zero?

   Answer: The coefficients must be opposites.

b. How are the new coefficients related to the coefficients in the *original* system?

   Answer: The new coefficients are multiples of the original coefficients.

   Teacher Tip: The idea of opposite coefficients is critical. The point is that the coefficients in the new system must be opposites that are divisible by the coefficients in the original system. Some students might make either *a* or *b* zero; if so, ask them to consider whether this will help them to solve the system.

c. Use the boxed equation to solve for *y,* and record your answer.

   Answer: Equations will vary, but regardless of the equation, *y* = 4. We can solve the resulting equation because it has only one variable, or in other words, one variable has been eliminated.

d. What is the solution to the system of equations? How do you know?

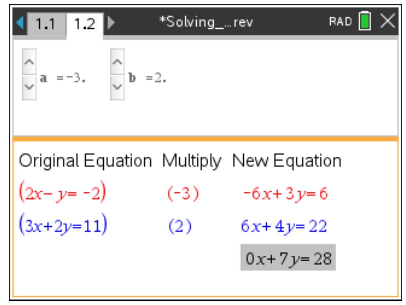
   Answer: The solution is *x* = 1 and *y* = 4 or {(1, 4)}. Substitute the *y* = 4 in one of the original equations. 2*x* – (4) = –2; *x* = 1.

   Teacher Tip: Students should check their solution by substituting their solution in the other equation.
   3*x* + 2*y* = 11: 3(1) + 2(4) = 11

4. Change the multipliers *a* and *b* to find different values that still produce a zero coefficient of *x* in the boxed equation. Record your multipliers:

   a =__________, *b* = _________

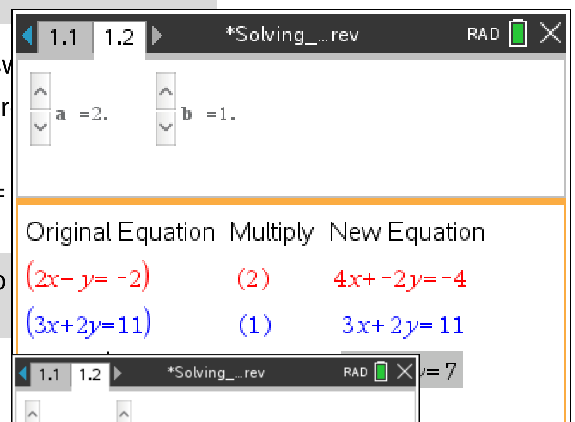
   Answer: Answers will vary: example *a* = –3 and *b* = 2.



a. How are the *new* coefficients related to the coefficients in the *original* system?

   Answer: The coefficients must be opposites. The new coefficients are divisible by the original coefficients.

   > **Teacher Tip:** The concept of opposite coefficients is critical. Reinforce this by having different students share different values for *a* and *b* that result in opposite coefficients for *x*. Discuss the fact that there are infinitely many such possibilities, but it is usually easiest to use the least common multiple (or a small common multiple—the lcm is not necessary). You might even suggest that the students use "reverse coefficient" multipliers with opposite signs, as appropriate. For example, if the coefficients of *x* are 5 and 2, you can use –2 and 5 (or 2 and –5) as multipliers. This would even work with 4 and 2 because you don't need to use the lcm (of course, they could multiply the second equation by –2). Students need to realize that any common multiple will work.

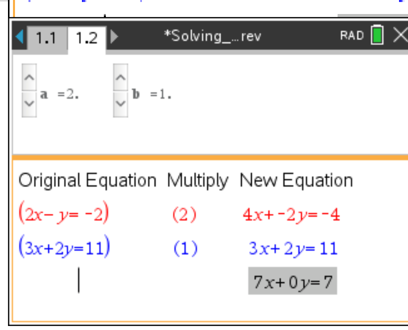


b. Use the new boxed equation to solve for *y*, and record your answ
   observe about this solution compared to the one you found in Pr

   Answer: Equations will vary, but regardless of the equation, *y* =

   > **Teacher Tip:** The point is that no matter how you choose to
   > you will always get the **same** solution for *y*.

5. Now change the multipliers *a* and *b* until the coefficient of *y* in the boxed equation is zero.



a. What do you observe about the coefficients of y in the *new* system?

Answer: The coefficients must be opposites. The new coefficients are divisible by the original coefficients.

b. Use the boxed equation to solve for x, and record your answer.

Answer: Equations will vary, but regardless of the equation, $x = 1$.

c. What is the solution to the system of equations, and how do you know?

Answer: Substitute $x = 1$ in one of the original equations
$2(1) – y = {}^{-}2$; $y = 4$; Solution is {(1, 4)}.

d. How does this compare to your solutions in Problems 3 and 4? Explain why.

Answer: The result is the same. The systems are equivalent, so they will have the same solution.

6. Is it possible to eliminate one variable by producing a coefficient of zero for x in any system of equations? Justify your answer.

Answer: Yes, it is always possible to do this because there are infinitely many common multiples of the original equations. There is always a value of b so that for any a times the coefficient of x in the first equation, b times the coefficient of x in the second equation is the opposite.

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Have students enter Yes or No in Quick Poll. Before revealing the results, ask if anyone found an example where this was not true.

Based on what you have learned, complete the following problems with pencil and paper:

7. Sarah was solving the system:
$–x + 4y = 8$
$–3x + 2y = 18$
She found $x = 4$. What should she do next, if anything?

Answer: The next step would be to substitute $x = 4$ into one of the original equations and solve for y.

TI-Nspire Navigator Opportunity

Have students add two Notes Pages to the .tns file. Students should press doc▾ **> Insert > Notes** to add page 1.3. Students repeat this process to add page 1.4. Students then record their solution to problem 7 on page 1.3 and their solution to problem 8 on page 1.4. When you are ready to discuss problem 7, have students go to page 1.3 and use Screen Capture to show all student solutions. Repeat for problem 8 and page 1.4. It may be possible to sort the screens into groups according to method used, misunderstandings, and so on.

8. Given the system:

$$3x + y = 7$$
$$5x - 2y = 8$$

a. Show how you would eliminate x.

Answer: Answers will vary. Example: Multiply the first equation by 5 and the second equation by –3. The resulting system would be: $15x + 5y = 35$ and $-15x + 6y = -24$.

b. Show how you would eliminate y.

Answer: Answers will vary. Example: Multiply the first equation by 2 and the second equation by 1. The resulting system would be: $6x + 2y = 14$ and $5x - 2y = 8$.

c. What is the solution to the system?

Answer: The solution to the system is $x = 2$ and $y = 1$ or $\{(2, 1)\}$.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students are able to understand:

- Equivalent equations can be generated by multipliers.
- Equivalent systems are composed of equivalent equations.
- A solution to a system of equations in two variables is an ordered pair of numbers that satisfies each equation in the system and can be found using elimination.