Transformations of Exponential Functions Part 2

Math Objectives

- Students will explore the family of exponential functions of the form $f(x) = b^{a \cdot x} + c$ and be able to describe the effect of each parameter on the graph of $y = f(x)$.
- Students will be able to determine the equation that corresponds to the graph of an exponential function.
- Students will understand that a horizontal dilation of the graph of an exponential function and a change of base of an exponential function are essentially the same.
- Students will look for and express regularity in repeated reasoning (CCSS Mathematical Practice).
- Students will look for and make use of structure (CCSS Mathematical Practice).

Vocabulary

- exponential function
- translation
- horizontal dilation
- parameter
- reflection
- change of base

About the Lesson

- This lesson involves the family of exponential functions of the form $f(x) = b^{a \cdot x} + c$.
- As a result students will:
  - Manipulate parameters, and observe the effect on the graph of the corresponding exponential function.
  - Conjecture and draw conclusions about the effect of each parameter on the graph of the exponential function.
  - Compare horizontal dilations and change of base and manipulate equations to demonstrate they are the same.
  - Match specific exponential functions with their corresponding graphs.

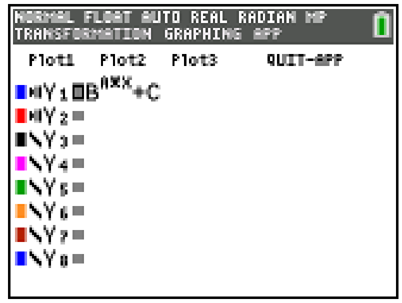
Teacher Preparation and Notes.

- This activity is done with the use of the TI-84 family as an aid to the problems.

Activity Materials

- Compatible TI Technologies: TI-84 Plus*, TI-84 Plus Silver Edition*, TI-84 Plus C Silver Edition, TI-84 Plus CE

** with the latest operating system (2.55MP) featuring MathPrint™ functionality.*

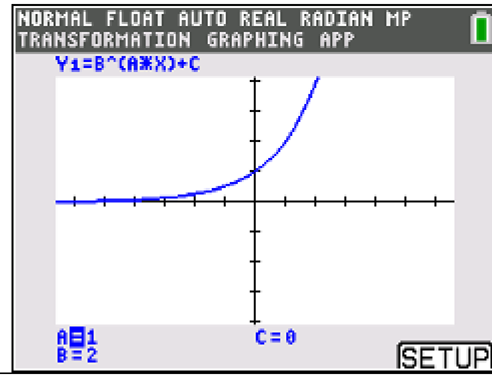


Tech Tips:

- This activity includes screen captures taken from the TI-84 Plus CE. It is also appropriate for use with the rest of the TI-84 Plus family. Slight variations to these directions may be required if using other calculator models.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:
Student Activity
Transformations of Exponential Functions_Part_2_84_Student.pdf
Transformations of Exponential Functions_Part_2_84_Student.doc

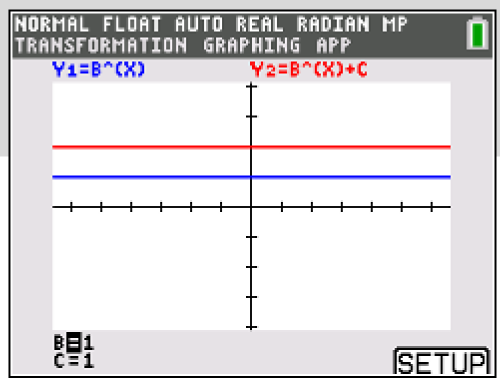
In this activity, you will examine the family of exponential functions of the form $f(x) = b^{a \cdot x} + c$ where a, b, and c are parameters. You will use the **Transformation App** (Transfrm) on your handheld to manipulate these parameters in Questions 1 - 3.



Discussion Points and Possible Answers

Tech Tip: To change the parameters throughout this activity using the **Transformation App** on the handheld, you will be using the arrow keys. Up and down move you from parameter to parameter, left and right change the value of each parameter. While on the graph, you can press Setup (graph) to manually change the parameters, including using decimals, or just type in the number you want while your cursor is on the parameter. You can use up to two functions in the app and they can be typed into Y_1 or Y_2.

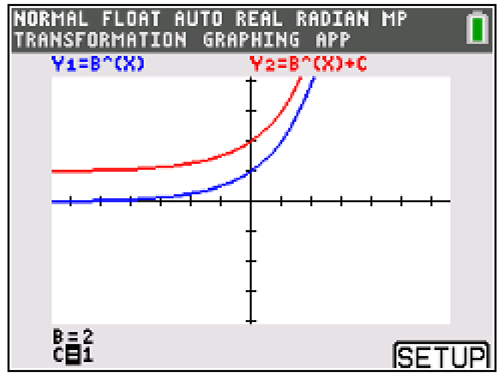
Teacher Tip: The default setting has B = 1. Since B is the base, the value needs to be something other than B = 1 as students explore the relationships in **Questions 1 - 3**.



The parameter b is the base of the exponential function and $b > 0, b \neq 1.$ Using the transformation application change the value of a parameter by entering the equation for each question into Y_1 or Y_2, and press the arrow keys to manipulate each parameter of the function on the graph.

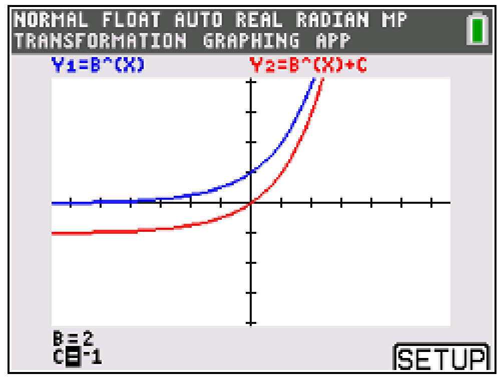
Question 1

Graph the following functions: $Y_1 = B^x$ and $Y_2 = B^x + C$. For specific values of B $(B \neq 1)$, press the arrows to change the value of C, and observe the changes in the graph of Y_1.



a. Explain why for every value of B the graph of Y_2 passes through the point $(0, C+1)$.

Sample Answer: The graph of $y = B^x$ passes through the point (0,1) for all values of $B > 0$ because $B^0 = 1$. The graph of $y = B^x + C$ is the graph of $y = B^x$ with a vertical translation of C units and when $x = 0$, $B^0 + C = C + 1$.



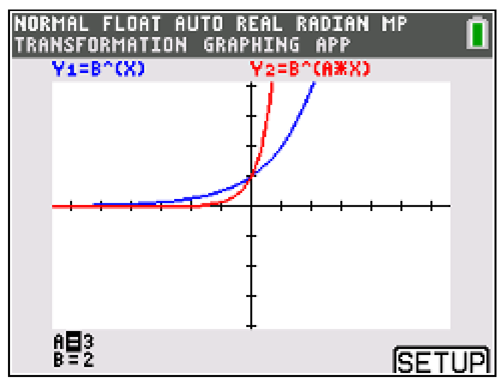
b. Is it possible for the graph of $Y_2 = B^x + C$ to intersect the x-axis? Explain why or why not.

Answer: The x-axis, the line $y = 0$, is a horizontal asymptote to the graph of $Y = B^x$. If the graph of the function has a vertical translation of $-C$ units, the graph of the function would intersect the x-axis.

Possible example: $y = 2^x - 1$

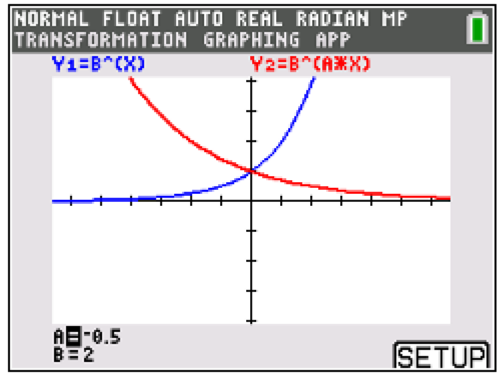
Question 2

Graph the following function: $Y_2 = B^{A \cdot x}$. For a specific value of B, click the arrows to change the value of A, and observe the changes in the graph of Y_1. Repeat this process for other values of B. Describe the effect of the parameter A on the graph of $Y_2 = B^{A \cdot x}$. Discuss the effects of both positive and negative values of A.



Answer

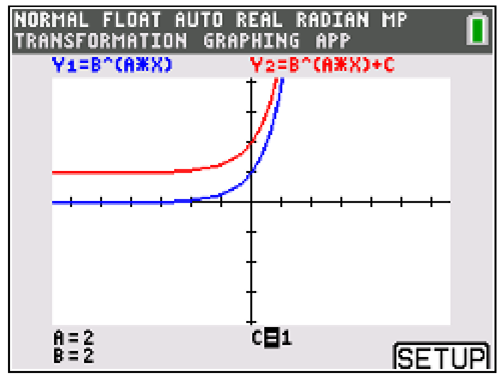
The graph has a horizontal dilation. For $|A| > 1$, the graph of $y = B^x$ is compressed horizontally by a factor of $\frac{1}{A}$. For $|A| < 1$, the graph of $y = B^x$ is stretched horizontally by a factor of $\frac{1}{A}$. If $A < 0$, , the graph is reflected across the y-axis..

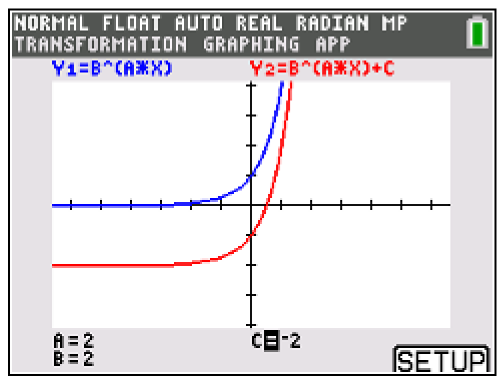


Question 3

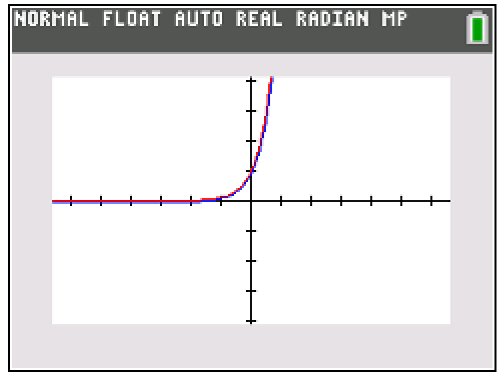
Graph the following functions: $Y_1 = B^{A \cdot x}$ and $Y_2 = B^{A \cdot x} + C$. For specific values of A and B, click the arrows to change the value of C, and observe the changes in the graph of Y_1. Describe the effect of the parameter C on the graph of $Y_2 = B^{A \cdot x} + C$. Discuss the effects of both positive and negative values of C.

Answer: The graph has a vertical translation. For $C > 0$, the graph of $y = B^{A \cdot x}$ is translated up. For $C < 0$, the graph of $y = B^{A \cdot x}$ is translated down.





Question 4



Turn off the Transformation App by selecting Quit-App on the $y =$ screen. Graph each function given and answer the following questions.

a. Display the graphs of $Y_1 = 3^{2x}$ and $Y_2 = 9^x$.

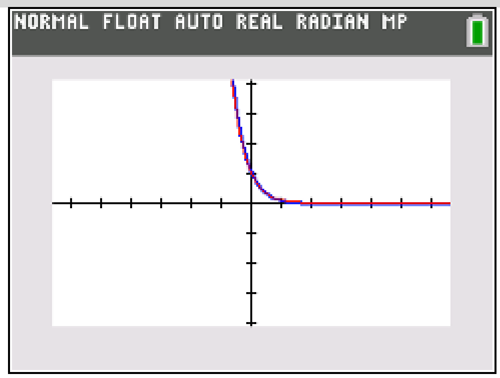
(i) How is the graph of Y_2 related to the graph of Y_1?

Answer: The graphs of these two exponential functions are the same.

(ii) Use the properties of exponents to justify your answer.

Answer: $Y_1 = 3^{2x} = \left(3^2\right)^x = 9^x = Y_2$

Teacher Tip: The thickness of the second function could be changed to thin so that both graphs are visible.



b. Display the graphs of $Y_1 = 3^{-2x}$ and $Y_2 = \left(\frac{1}{9}\right)^x$.

(i) How is the graph of Y_2 related to the graph of Y_1?

Answer: The graphs of these two exponential functions are the same.

(ii) Use the properties of exponents to justify your answer.

Answer: $y_1 = 3^{-2x} = \left(3^{-2}\right)^x = \left(\frac{1}{9}\right)^x = Y_2$.

c. Use your answers to parts (a) and (b) to explain the relationship between a horizontal dilation of the graph of an exponential function and a change of base of the exponential function.

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Answer: A horizontal dilation of the graph of an exponential function and a change of base are essentially the same. Consider the following expression to show this analytically.

$$f(x) = B^{A \cdot x} = \left(B^A\right)^x = C^x,$$

where $B^A =$ a constant, and $A \neq 0$. This demonstrates that any horizontal dilation can also be considered a change of base of exponential functions.

Question 5

5. Without using your calculator, match each equation with its corresponding graph.
Check your answers by graphing each function on your calculator.

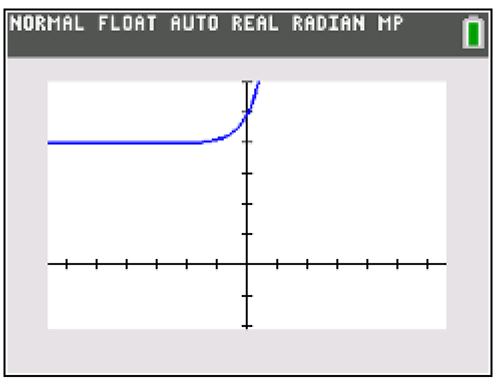
(a) $f(x) = 2^{3x}$ (b) $f(x) = -(2)^{3x}$

(c) $f(x) = 2^{-3x}$ (d) $f(x) = 2^{3x} + 4$

(e) $f(x) = e^{-x}$ (f) $f(x) = e^{x} - 3$

Note: The function in part (e) is the "natural" exponential function and involves the number $e \approx 2.71828...$

(i)



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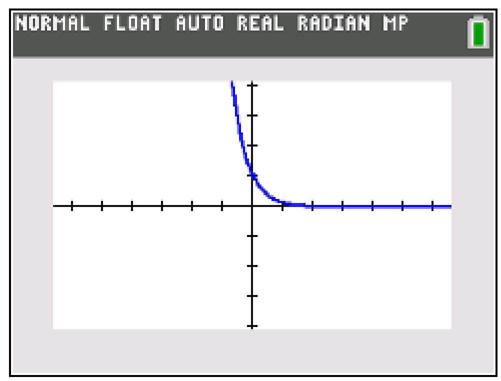

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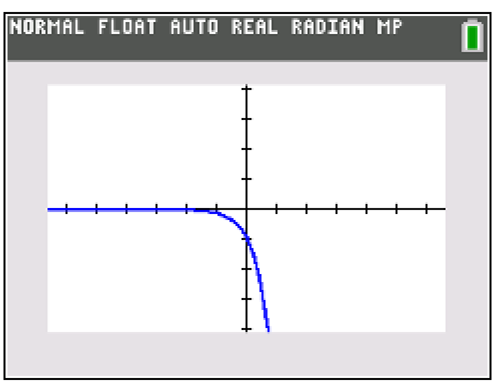

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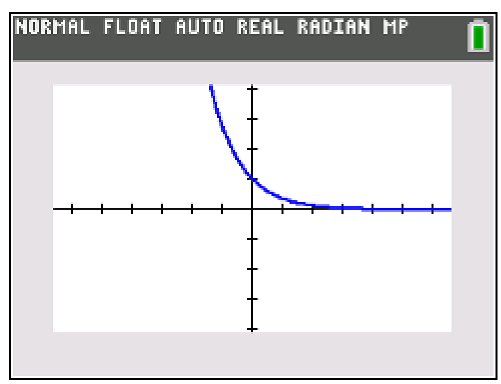

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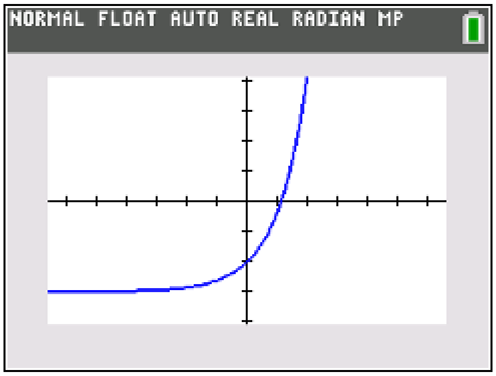
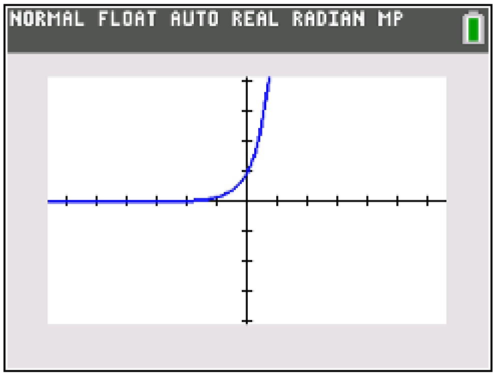


(vi)



Answers: (a) (vi); (b) (iii); (c) (ii); (d) (i); (e) (iv); (f) (v).

Extensions

1. Ask students to display the graph $f(x)=3^{2x}-2$ and to find the range.
2. Ask students to display and compare the graphs of $f(x)=\left(\frac{1}{3}\right)^{-2x}$ and $f(x)=-\left(\frac{1}{3}\right)^{2x}$.

Wrap Up

Upon completion of the lesson, the teacher should ensure that students are able to understand:

- How to graph an exponential function of the form $f(x)=b^{ax}+c$.
- How to explain the concepts of dilations and translation.