What is Log?

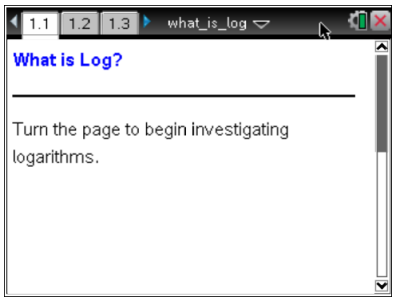
Student Activity

Name _____

Class _____

Open the TI-Nspire document *What_is_Log.tns.*

You may have noticed that above 10^x is $\log$. What does *log* mean? Why is $\log$ placed above an exponential key? You will investigate these questions in this activity.



Move to page 1.2.

1. The graph of the function $f(x) = 2^x$ is shown.
   a. What are the domain and range of $f(x)$?

   b. Recall that $f(x) = 2^x$ is a one-to-one function, so it has an inverse reflected over the line $y = x$. What are the domain and range of $f^{-1}(x)$?

   c. Point P is a point on $f(x)$. Move the Show Reflection slider to *Yes* to and then move point P. As you do so, point P' invisibly traces the graph of $f^{-1}(x)$. Since $f(x)$ can be written as $y = 2^x$, write a corresponding equation for the inverse.

   d. The equation $x = 2^y$ cannot be written as a function of y in terms of x without new notation. Move the Show Function slider to *Yes*. The inverse of $f(x)$ is actually $f^{-1}(x) = \log_2(x)$. In general, $\log_b x = y$ is equivalent to $b^y = x$ for $x > 0$, $b > 0$ and $b \neq 1$. Why do you think x and b must be greater than 0? Why can b not be equal to 1?

e. Move point *P so* that its coordinates are (1, 2). The point (1, 2) on $f(x) = 2^x$ indicates that $2^1 = 2$. *P′* has the coordinates $(2, 1)$. The point (2, 1) on $f^{-1}(x) = \log_2(x)$ indicates that $\log_2 2 = 1$. Use this relationship between exponential expressions and logarithmic expressions to complete the following table. (Move point *P* as necessary.)

P	*P′*	Exponential Expression	Logarithmic Expression
(1, 2)	(2, 1)	$2^1 = 2$	$\log_2 2 = 1$
(2, 4)			
	(8, 3)		
		$2^0 = 1$	
		$2^{-1} = \frac{1}{2}$	
$\left(-2, \frac{1}{4}\right)$			
			$\log_2 \frac{1}{8} = -3$

2. You have discussed the idea of reflecting the exponential function over the line $y = x$. The result of this reflection is the logarithmic function. Now we will discuss any tabular relationships that are formed between an exponential function and a logarithmic function.

Using the first and second columns from the table above, fill in the following tables.

x	$f(x) = 2^x$
-3	
-2	
-1	
0	
1	
2	
3	

x	$f^{-1}(x) = \log_2 x$

$1/8$	
$1/4$	
$1/2$	
1	
2	
4	
8	

(a) Briefly explain your process of filling in the tables on the previous page.

(b) With a classmate, discuss and describe the patterns you see in each individual column.

(c) Write down a rule for each table that you can use to classify the function as either exponential or logarithmic.

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3. Solve the logarithmic equation $\log_2 32 = y$ using the patterns from question 1. Then, use the slider to change the *n*-value to solve the logarithmic equation. How does the exponential equation verify your result?

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4. Solve the equation $\log_4 \frac{1}{256} = y$. Then, use the slider to change the *n*-value to solve the logarithmic equation. How does the exponential equation verify your result?

5. Maya solved the logarithmic equation $\log_4 16 = y$. She says the answer is 4 since $4 \times 4 = 16$. Is her answer correct? Why or why not?

6. Alex says that when solving a logarithmic equation in the form $\log_b a = y$, he can rewrite it as $b^a = y$. Is this a good strategy? Why or why not?