# *t* Distributions

**Student Activity**

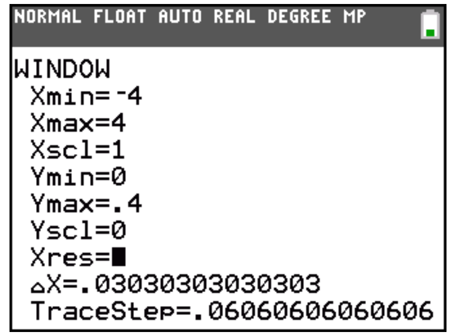
Name ___________________

Class ___________________

The *t* distribution is used when *n* is small (less than 30) and the population standard deviation is unknown. For a sample size of *n*, the number of degrees of freedom is *n* – 1.

**Problem 1 – Characteristics of the *t* Distribution**

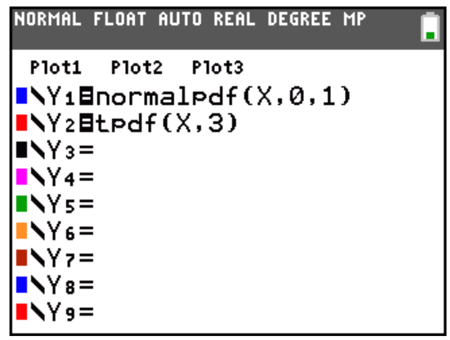
Press [WINDOW] and set the values as shown at the right.




Graph the standard normal distribution: **Y1** = **normalpdf(X, 0, 1)**.

Then graph the *t* distribution for *n* = 4: **Y2** = **tpdf(X, 3)**.

Note: The **normalpdf** and **tpdf** commands are in the Distribution menu ([2nd] [DISTR]).




The format is (*x*, degrees of freedom). To help distinguish this graph from the first, make it bold by moving the cursor to the left of **Y2** and pressing [ENTER] so that a bolder line appears.

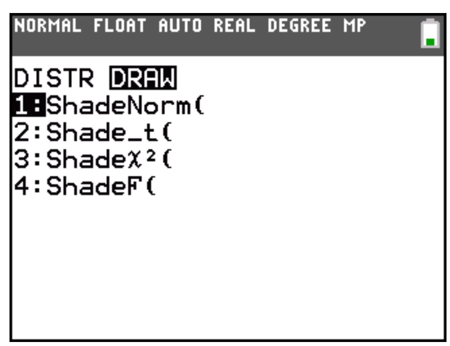
1. How does the *t* distribution for *n* = 4 (d.f. = 3) compare to the normal distribution?

Press [Y=] and change Y2 to a *t* distribution where *n* = 9 (degrees of freedom = 8). Press [GRAPH]. Repeat for *n* = 16 and *n* = 26.

2. What happens as *n* gets larger? Why?

**Problem 2 – Comparing Areas**

Press [Y=] and clear the entries. On the Home screen enter **ShadeNorm(–3, 3, 0, 1)** and press [ENTER] to find and display the area under the standard normal curve that is within three standard deviations of the mean.




**ShadeNorm** is accessed by pressing [2nd] [DISTR] and then moving to the **DRAW** menu. The format is (lower bound, upper bound, mean, standard deviation).

3. What is the value of this area?

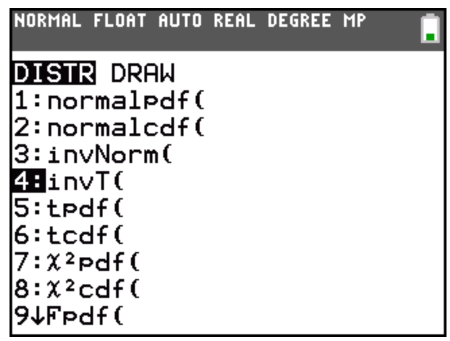
Press [2nd] [DRAW] [ENTER] to clear the drawing. On the Home screen enter **Shade_t(–3, 3, 3)** and press [ENTER] to find and display the area between these same points under the *t* distribution.
Note: **Shade_t** is in the **DRAW** menu. The format is (lower bound, upper bound, d. f.).

**4.** What is this area?

**5.** In the same way, find the area under the *t* distribution for 8, 15, and 25 d. f.

**6.** What happens to the area and why?

**Problem 3 – Critical Values for a *t* Distribution**




To find a critical value for a *t* distribution, use the **invT** command located in the **DISTR** menu. Similar to **invNorm**, **invT** will give the *t*-value associated with a given area to the left of that value. The format for **invT** is (area to the left, degrees of freedom). The format for **invNorm** is (area to the left, mean, standard deviation).

**7.** Verify that $t_{\frac{\alpha}{2}} \approx 4.303$ for $n = 3$ at the 95% level. Then complete the chart by finding each value at the 95% level.

| $t_{\frac{\alpha}{2}}$, $n = 3$ | $t_{\frac{\alpha}{2}}$, $n = 8$ | $t_{\frac{\alpha}{2}}$, $n = 15$ | $t_{\frac{\alpha}{2}}$, $n = 25$ | $z_{\frac{\alpha}{2}}$ |
|---|---|---|---|---|
| 4.303 | | | | |

**8.** If any, what patterns do you see?

**9.** Predict how the following will compare among each other.

50% CI, $t_{\frac{\alpha}{2}}$, $n = 4$ and $n = 28$

80% CI, $t_{\frac{\alpha}{2}}$, $n = 4$ and $n = 28$

100% CI, $t_{\frac{\alpha}{2}}$, $n = 4$ and $n = 28$

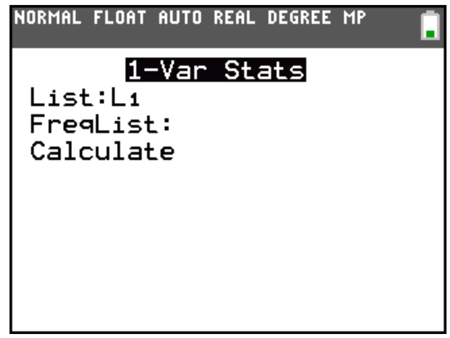
**10.** Find the six critical values listed above by using the **invT** command.

**Problem 4 – Constructing a Confidence Interval**

For a *t* distribution, the margin of error for estimating the population mean is given by $E = t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}}$.

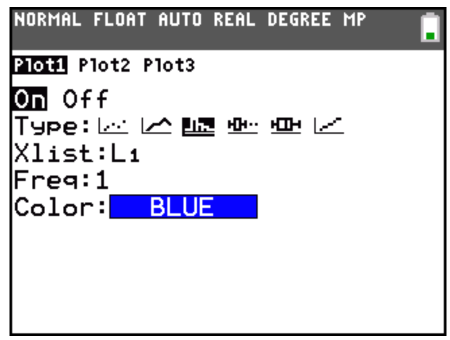
The weights of 10 randomly selected newborn kittens, in grams, are shown below. Enter these values into **L1** by pressing STAT and choosing **Edit**.

98, 107, 101, 102, 94, 103, 105, 97, 99, 102




Press STAT, arrow to the **CALC** menu, choose **1-Var Stats**, and enter **L1**.

**11.** What is the mean and standard deviation of the weights?




Graph the weights to verify that the distribution is roughly normal. Press 2nd [STAT PLOT] and select **Plot1**. Match the settings as shown at the right.

Press ZOOM and choose **ZoomStat** to get an appropriate viewing window.

**12.** Calculate a 90% confidence interval and a 95% confidence interval for the mean weight of all newborn kittens.

90%: critical value: __________, margin of error: _________, confidence interval: _________

95%: critical value: __________, margin of error: _________, confidence interval: _________

Ten *more* newborn kittens are randomly selected and weighed. Their weights, in grams, are

97, 104, 92, 96, 100, 105, 103, 95, 92, 109

**13.** Add these weights to list **L1**. What is the new mean and standard deviation?

**14.** Calculate a new 90% confidence interval and a 95% confidence interval for the mean weight of all newborn kittens. ($n$ = 20)

90%: critical value: _________, margin of error: ________, confidence interval: _______

95%: critical value: _________, margin of error: ________, confidence interval: _______

Check your confidence intervals by pressing STAT, moving to the **TESTS** menu, and selecting **TInterval**. You have the option of choosing data from a list (choose **Data**) or by entering the sample mean, standard deviation, and sample size (choose **Stats**).