

Pendulum Swing

Answers – Part 1

7 8 9 10 11 12



Introduction

The periodic motion of a pendulum provides enjoyment for a child sitting on a swing, in a clock it provides a means of measuring time and for ancient scientists, proof the earth rotates on its axis. For these and many other reasons a mathematical investigation into the motion a pendulum is a valuable experience.

Consider a child's swing, at the highest point the rider is momentarily at rest, they descend and accelerate. At the base of the swing the child reaches a maximum speed after which they decelerate and eventually come momentarily to rest once again. The return swing is a reversal of the above, and repeated swings an example of 'periodic' motion.

The position, velocity and acceleration changes are all related and can be modelled mathematically. In this investigation position and time data for a pendulum will be modelled using a trigonometric function.

Equipment

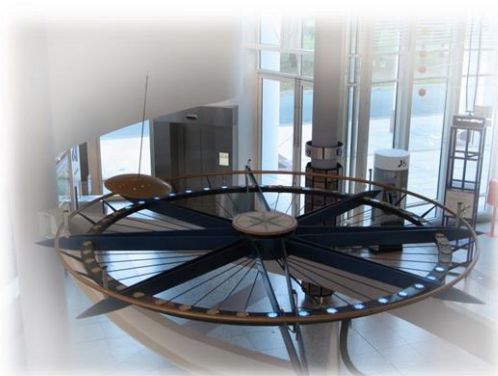
- TI-nspire calculator or computer software
- CBR2 motion sensor and connectivity cable
- Pendulum – 30cm polystyrene fishing float. (Recommended)

Collecting the data

Read through the instructions and Preliminary Question set **before** collecting data.

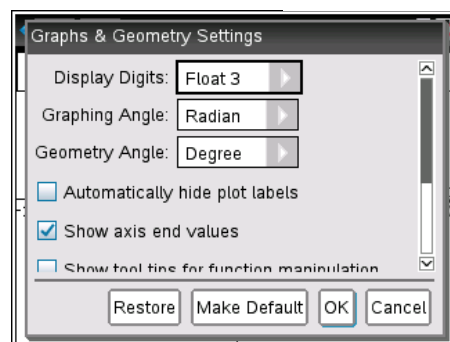
Start a new TI-Nspire document, insert a Graphs application.

Make sure the document settings are for angles to be measured in radians in the Calculator and Graphs applications.



Foucault Pendulum – Questacon (Canberra)

A pendulum swings in a single plane. As the earth rotates the pendulum *appears* to change direction (precession), in fact it continues in its original plane thus illustrating the rotation of the earth. The precession of the pendulum can be used to determine the location (latitude only) of the pendulum.

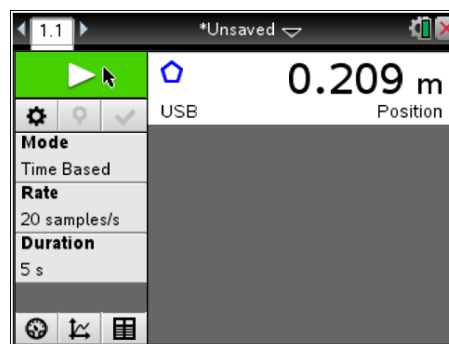


Connect the CBR2 motion sensor using the connectivity cable, the Vernier DataQuest™ application will automatically launch.

The default sampling rate for the motion sensor is generally ideal for a pendulum swing. Try and collect at least three complete swings of the pendulum.

Aim the CBR2 at the pendulum and set the pendulum in motion so that it swings backwards and forwards, away from and towards the sensor.

Use the touchpad to move the mouse over the start button and click to start recording.



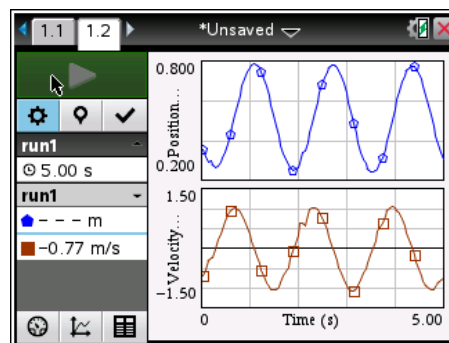
The Tab and Enter keys are efficient ways to navigate the DataQuest menus and settings.

A clicking sound indicates the sensor is active; the ultrasonic waves generated and used by the motion sensor are inaudible.

When data *collection* is active the clicking sound occurs more frequently.

If the first data sample is not satisfactory, touch the start button once again to collect another sample. The screen shown opposite provides an indication of *reasonable* data.

Once data collection is complete the cable can be removed from the calculator.



Preliminary Questions – (Read before commencing data collection)

1. **Estimate** the straight line distance between the two extreme points of the pendulum's swing during data collection. (Answer in metres)

Answers will vary; typically the peak to peak distance should be within the range: 0.2m to 1m. The question is designed to *help* students determine parameter *a*, where *a* will be half this value.

2. During data collection, **estimate** the rest (mean) position of the pendulum relative to the motion sensor. (Answer in metres)

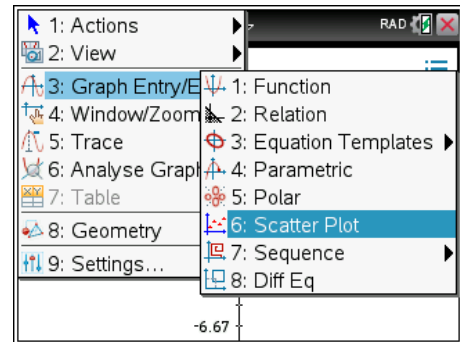
Answers will vary; typically the pendulum starts somewhere within the range: 0.4m to 1.8m. This question is designed to *help* students determine parameter *c*.

3. Approximately how long does it take for the pendulum to complete one to and fro swing. The average of multiple swings will help produce a more accurate answer. (Answer in seconds)

Answers will vary; the default time setting on the motion detector is for 5 seconds of data collection. Averaging the time over 10 seconds (or for 10 swings) generally provides an estimate within 10% of the final required value. Question designed to *help* students determine parameter *b*.

Graphing the Data

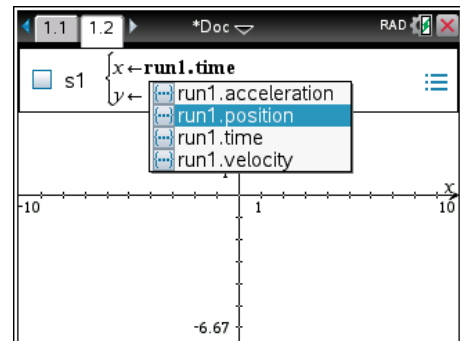
Insert a **Graph** application and change the **Graph Entry** type to a **Scatter Plot**.



When the cursor is adjacent to the x variable, press the [VAR] button to display stored variables. (As shown)

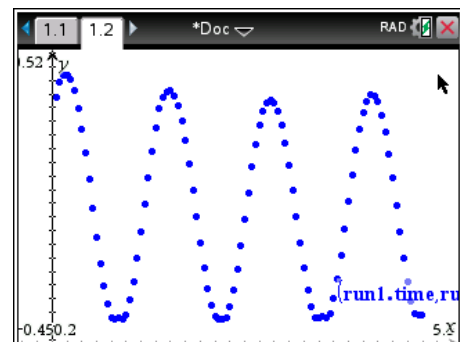
Set up a scatter plot with:

- X axis = **run1.time** (time)
- Y axis = **run1.position** (position)



The data is initially displayed on the default window setting.

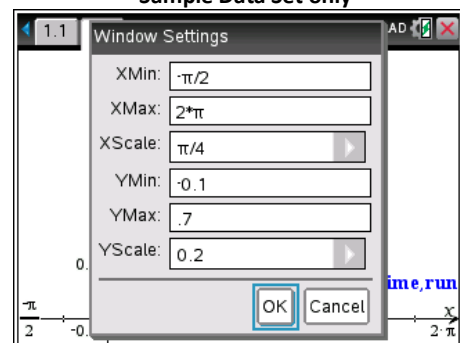
Use the Menu to change the **Window** settings and select **Zoom Data**.



Sample Data Set only

The data and subsequent graphs will make more sense when the x axis (time) is visible. From the menu select **Window Settings** and adjust them using the values shown opposite as a guide.

Actual settings will depend on your data. Including a negative value for both the x-min and y-min values means the axis will both be visible.



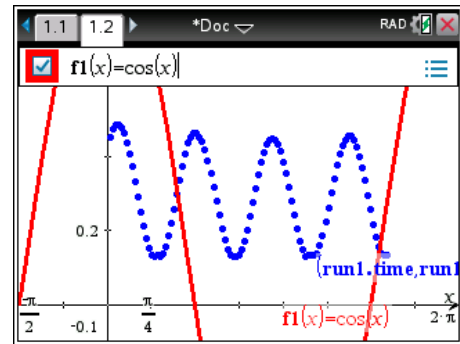
Determining an Equation

Change the **Graph Entry** back to a **function**.

In the function line enter the equation:

$$f_1(x) = \cos(x)$$

In the absence of any transformations the cosine function does not model the pendulum data very well at all!



Insert a Calculator application and calculate the following:

Mean(run1.position)



Use the **VAR** key to continue accessing the pendulum data and any other variables stored in the calculator.

Questions

4. Explain how the mean position relates to the pendulum's motion and any assumptions about this calculation.

The assumption here is that a whole number of to and fro swings has been measured or completed so that a periodically symmetrical sample of data points occur on either side of the rest position. Even if this is not the case, a sufficient number of completed swings will 'average' out some of the partially complete swing data. The mean or rest position can be used to determine the vertical translation (parallel to y axis) and also assist in the determination of the amplitude.



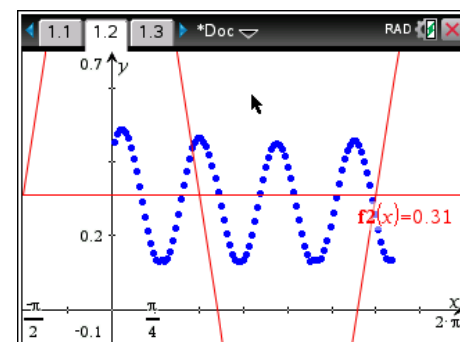
On the Graph application plot the mean position of the pendulum in $f_2(x)$. Adjust the value until it provides the best representation of the mean position of a complete to and fro pendulum swing.

5. Use your answers to questions 2 and 4 to help determine the appropriate parameter in the general equation for the pendulum:

$$f_1(x) = a \cdot \cos(b \cdot (x - h)) + c$$

Write down your equation and include a calculator screen image of your data and equation.

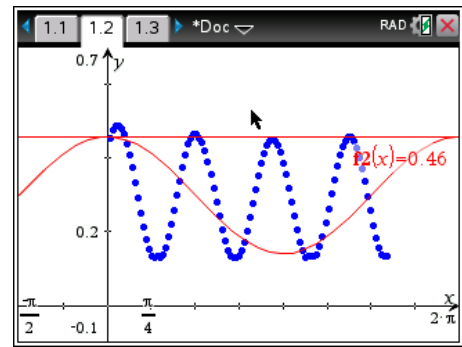
$$f_1(x) = \cos(x) + 0.31 \quad (\text{Answers will vary})$$



6. Use the calculator application to determine: $\text{Max}(\text{run1.position})$ and explain how this will *assist* in the determination of the equation for the pendulum. Graph this value in $f_2(x)$ and adjust it accordingly to best align with the graph peaks.

Adjust the equation in $f_1(x)$ to include the appropriate transformation. Write down your new equation and include a screen image of your data with your new equation.

$$f_1(x) = 0.15\cos(x) + 0.31 \quad (\text{Answers will vary})$$

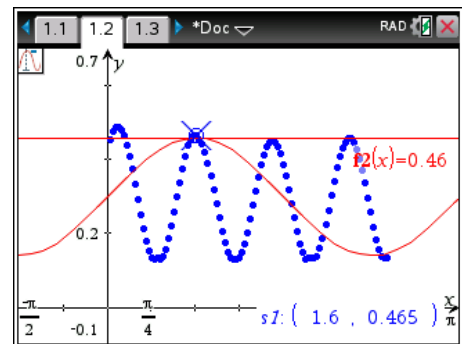


7. Use the **trace** feature in the graph application to explore the pendulum data. If the trace does not automatically go to the data points, use the up/down arrows on the navigation pad to toggle between graphs. Locate the coordinates of a maximum point on the pendulum data and explain how this will help determine another transformation for the selected function.

Adjust the equation in your calculator to include the appropriate transformation. Write down your new equation and include a screen image of your data with your new equation.

Note that in the example shown opposite, the first maximum point was not used, any maximum point can be used.

$$f_1(x) = 0.15\cos(x - 1.6) + 0.31 \quad (\text{Answers will vary})$$

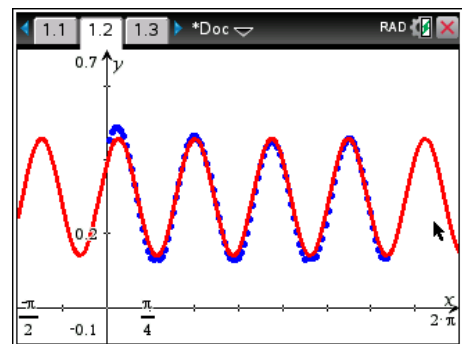


8. In question 3 an estimate for the time taken to complete one full swing was determined. Use this information to determine how many swings would take place in 6.3 seconds. ($\sim 2\pi$) Check your answer by tracing over the data to confirm that your estimate is correct.

Adjust the equation in your calculator to include the appropriate transformation and fine tune as necessary. Write down your new equation and include a screen image of your data with your new equation.

The data shown opposite includes approximately 3.5 swings in 5 seconds. This equates to approximately 1.4 seconds per swing, therefore $2\pi \div 1.4 \approx 4.4$. The final graph shown is:

$$f_1(x) = 0.15\cos(4.5(x - 1.6)) + 0.31 \quad (\text{Answers will vary})$$

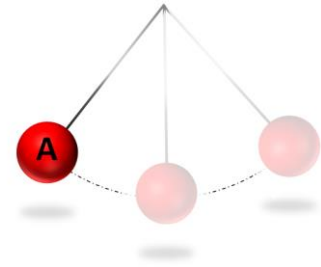


9. Fine tune your equation so that it matches your data and record your final equation. Discuss the relationship between the physical motion of the pendulum and the equation to your graph. Use the images and prompts provided to guide and support your discussion. For the purposes of this discussion imaging the motion detector is placed to the left of point A.

Position A: Highlight this location on your graph and relate the estimated dimensions in the preliminary questions to the final equation parameters.

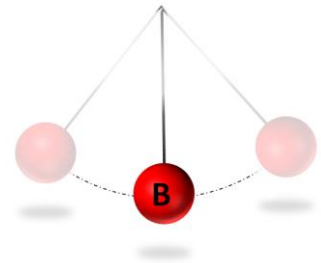
As the motion detector is just to the left of point A, this position would represent the minimum point on the cosine graph and the closest point of the pendulum to the motion detector.

This value could be obtained by using the mean rest position and maximum value. $(2B - A)$



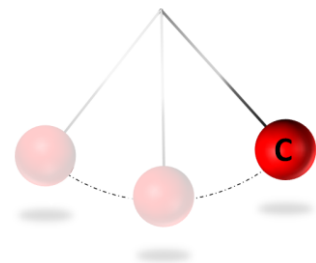
Position B: Highlight this location on your graph and relate the estimated dimensions in the preliminary questions to the final equation parameters. Include other characteristics such as the speed and direction of the pendulum at this point.

The movement direction is implied here, therefore students should highlight the inflection point with positive gradient; however in the absence of specific direction arrows, students may also highlight the inflection points with negative gradient.



Position C: Highlight this location on your graph, compare and contrast this with position A.

This point on the graph corresponds to the highest point on the curve (maximums). The physical symmetry of the pendulum's motion equates to a reflection of the curvature at point A.



10. In the calculator application define a function: $g(x) = \sin(x)$.
- If $g(x)$ is to model the same data set, which transformations can be used directly from the previous set?
The parameters: a , b and k will be the same. The horizontal transformation (h) will be different.
 - Determine the value(s) for the remaining parameter(s).
Answers will vary. In the example shown $h \approx 1.6 - \frac{1.4}{4}$ where 1.4 was determined as the approximate amount of time for 1 complete swing.
 - In the graph application use function notation ie: $f_2(x) = a \cdot g(b \cdot (x - h)) + k$ to check your answers and include a screen image showing the data and the new function only.
Answers will vary. The function $f_2(x)$ should be directly over $f_1(x)$ and the parameters a , b , and k should all be the same as for $f_1(x)$.