

# Introduction to Parabolas

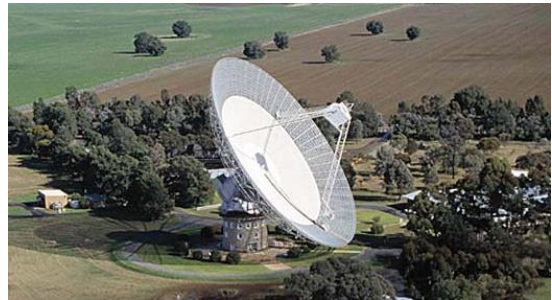
## Teacher Notes & Answers

7 8 9 10 11 12



## Introduction

Parabolas belong to a group of curves called “conic” sections. They are so named because they can be obtained by slicing a cone at various angles. Properties of a parabola make them particularly useful in the world around us particularly in satellite dishes, car headlights and other objects where bringing waves to a single focus or projecting them from a point is useful. Another conic shape is the ellipse, often loosely referred to as an oval. Aside from its obvious sporting applications, the ellipse is used in medical equipment and also describes the motion of planets.



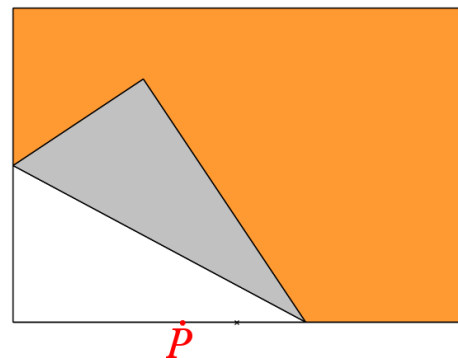
## Teacher Notes:

A selection of dynamic representations is contained within the Teacher Presentation (TNS) file that illustrates aspects of parabolas including:

- Paper Folding
- Class Locus
- Conic Sections

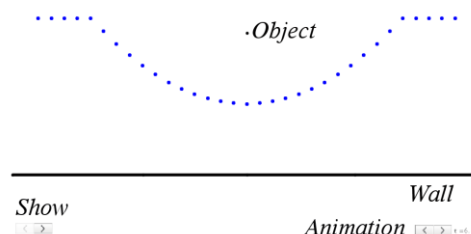
Open the file and set the document preview to “Computer”.

**Paper folding** is another way to introduce students to parabolas. On a blank A4 piece of paper, held in a ‘landscape’ orientation, students place a dot approximately 5 to 6 cm from the base of the page and 15cm from the side. Repeatedly fold the page so that the base of the page just touches the dot. Mark the crease formed by the fold.



A dynamic version of this is included in the Teacher Presentation (TNS) file. (See image opposite)

**Class Locus** – Have students stand along the back wall of the classroom. Place an object approximately 2 metres from the back wall but ‘centred’ across the room. Instruct students to walk away from the wall (perpendicular to the wall) until they are the same distance from the wall as they are object. The students form a locus. A dynamic illustration of this is also included in the Teacher Presentation (TNS) file. (Shown opposite) You can move the object and animate the students



(dots) to show their relative positions as they move away from the wall. A show/hide option also allows you to reveal how this is related to the paper folding task.

More dynamic diagrams are located on the Teacher Presentation TNS file.

## Creating an envelope

Open the TI-Nspire document: **"More than a Graph"**

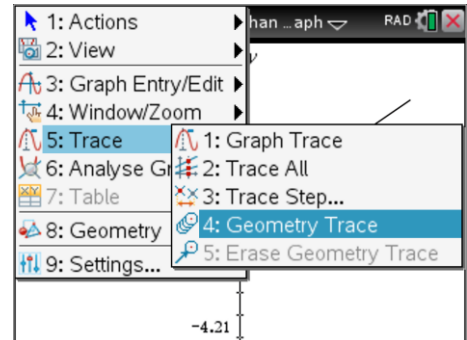
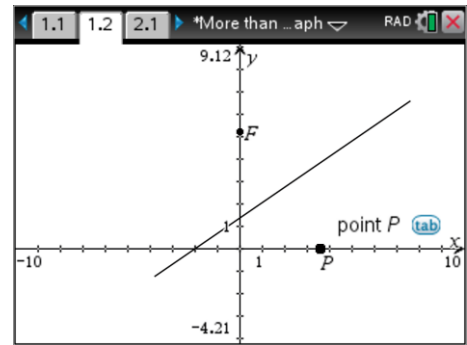
Read the instructions on Page 1.1 and then navigate to Page 1.2 where a geometric construction has been placed on a graph.

Use the mouse over point P then click and hold to grab the point. Move point P along the x axis and watch the path of the line. The line is a 'perpendicular bisector'. Each point on the line is same distance from P as it is from F (equidistant).

Imagine if the line left a residue to show where it has been. This can be done two ways.

[ Menu ] > Trace > Geometry Trace

Click on the perpendicular bisector (line) then grab point P and move it along the x axis.



### Question: 1

The series of lines left by the Geometry Trace forms an 'envelope'. What is the shape of the envelope? **Parabola**

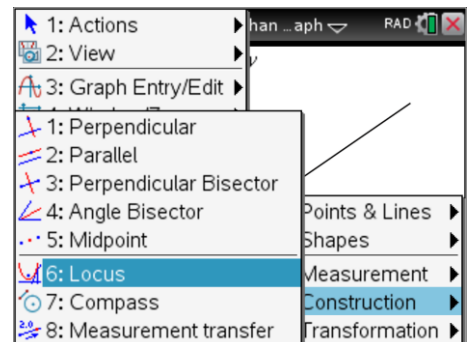
The geometry trace can be cleared off by using the "Erase Geometry Trace" option.

[ Menu ] > Trace > Erase Geometry Trace

A dynamic trace can be constructed using the Locus tool.

[ Menu ] > Geometry > Construct > Locus

Click on the line followed by Point P.



### Question: 2

Once the Locus has been constructed, grab point F and move it up and down the y axis. How does moving point F change the shape of the envelope? **Point F 'dilates' the parabola and also a 'translation'.**

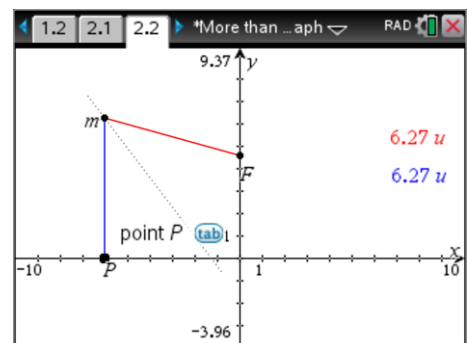
Navigate to page 2.2. Grab point P and observe the position of point M and its distance from P and F.

The dotted line is a perpendicular bisector of points P and F.

A locus can be constructed on point M as point P moves along the x axis.

[ Menu ] > Geometry > Construct > Locus

Select point M and then point P to see the locus.



**Question: 3**

What shape is the locus? **Parabola**

**Question: 4**

If point F is at (0, 4) where would the locus cross the y axis? **(0, 2) – half way between directrix and F.**

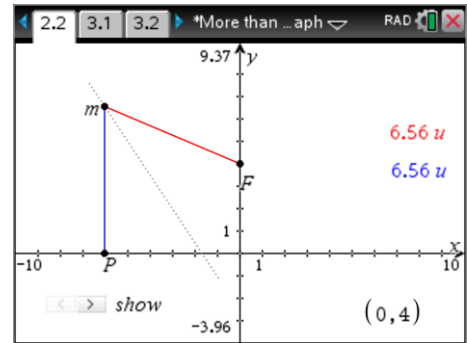
Navigate to page 2.2. Grab point P and observe the position of point M and its distance from P and F.

The dotted line is a perpendicular bisector of points P and F.

A locus can be constructed on point M as point P moves along the x axis.

[ Menu ] > Geometry > Construct > Locus

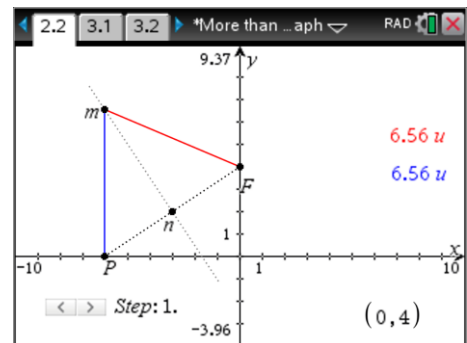
Select point M and then point P to see the locus.

**Question: 5**

What shape is the locus? **Parabola – same construction as previous.**

The slider in the bottom left corner of the screen can be used to reveal how the perpendicular bisector helps construct the shape of the locus.

Click on the slider to see Steps 1 through to 3.

**Question: 6**

Given that n is the midpoint of PF, describe the relationship between  $\triangle MNP$  and  $\triangle MNF$  and therefore the relationship between lengths MP and MF.

**Hint:** The slider can help reveal the details.

**PN = FN (Perpendicular bisector so N is the midpoint of PF.)**

**MN is common**

**$\angle PNM = \angle FNM$  (Perpendicular)**

**$\triangle MNP \cong \triangle MNF$  (SAS)**

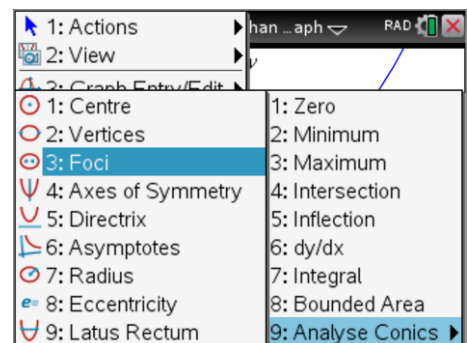
Navigate to page 3.2 where a parabola and its corresponding equation are displayed. This time the calculator will do the geometry.

[ Menu ] > Analyse Graph > Analyse Conics > Foci

Select the parabola and the 'focus' and its coordinates appear.

[ Menu ] > Analyse Graph > Analyse Conics > Directrix

Select the parabola and a line called the Directrix will appear.



The focus equates to point F in the previous diagrams. The x axis was used as the directrix in previous diagrams.

**Question: 7**

Compare the distance from the focus to a point P on the parabola and the distance from P to the directrix.

The distance is the same from focus to curve and curve to directrix. (Definition of a parabola)

**Application**

The radio telescope at Parkes has a 'diameter' of 64metres. When radio waves bounce off the surface they are reflected back towards the focal point 27metres away. Imagine the surface of the dish as a two dimensional space.

**Question: 8**

Determine the area of this space in square centimetres ( $\text{cm}^2$ ).

$$\begin{aligned} A &= \pi r^2 \\ &= 3200^2 \times \pi \\ &\approx 32,169,908\text{cm}^2 \end{aligned}$$

**Question: 9**

Suppose the dish is accurate enough to focus these waves into a region that is only  $10\text{cm}^2$ . The energy from the waves collected by the entire dish now fit into this region. Calculate the increase in intensity.

(Intensity: Overall area of dish divided by focused area.)

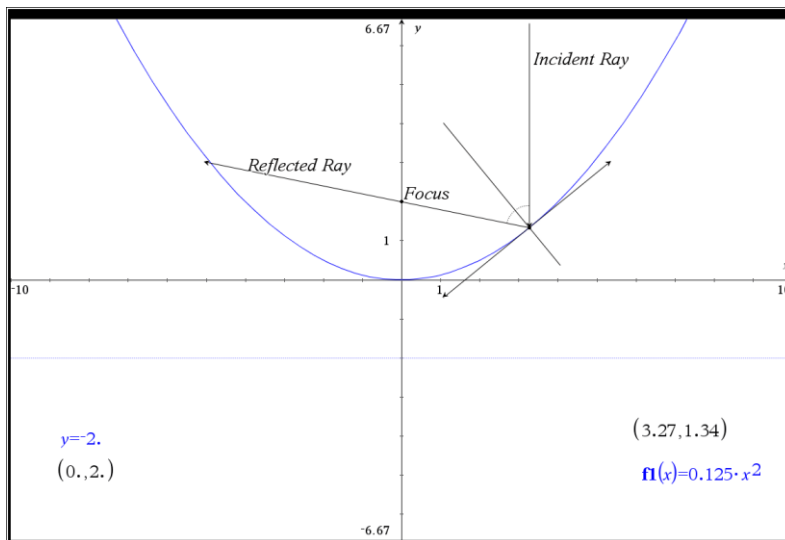
Answer: The intensity will increase by a factor of approximately 3,216,991 times.

No wonder it can pick up signals from more than 20,000,000 light years away  $\approx 189,214,000,000,000,000\text{km}$ !

**Teacher Notes:**

Problem 3 of the Teacher Presentation TNS file contains a dynamic representation of the 'practical' properties of a parabola that are used in light reflectors and telescopes such as the one located at Parkes.

Drag the point on the parabola around to see how the reflected light ray always passes through the focus. The incident ray is parallel to the 'principle axis'. For radio telescopes such as the one located in Parkes (NSW), radio waves enter from space. As their source is a long way away, the waves are essentially travelling in the direction of the principle axis (y axis) and therefore reflect through the focus. The focus can be seen in the picture of the Parkes telescope as the 'box' supported above the parabola.



A step by step derivation of the equation for a parabola is in **Problem 4** of the Teacher Presentation TNS file.

From the activity, students should identify that distance  $PF = PD$ .

**Distance PF** can be computed using Pythagoras's theorem.

$$FQ = x$$

$$PQ = y - f \text{ (where } f \text{ is the } y \text{ coordinate of the focal point.)}$$

$$PF = \sqrt{x^2 + (y - f)^2}$$

**Distance PD** =  $y + d$  (where  $d$  is the directrix location)

**Solve:** Students can use the solve command to make  $y$  the subject of the equation:

$$y = \frac{x^2 + f^2 - d^2}{2(f - d)}$$

Substitute  $d = -1/4$  and  $f = 1/4$  results in the equation  $y = x^2$ .

The above equation is for all parabolas with a vertex located on the  $y$  axis. Other equations can be determined by providing translations of the point  $f$  and directrix  $d$ .

The focus of the Parkes telescope is 27m. The dish has a 'diameter' of 64m. Define this as a parabola passing through the origin with appropriate domain restrictions.

Have students determine the equation to more parabolas given the focus and directrix, check the equations using the software.

A three dimensional cone and plane are located in Problem 4. The values of  $a$ ,  $b$  and  $c$  can be changed to alter the cut through the cone. Drag the axis around to rotate the graph or select the graph and use the left/right and up/down arrows on your computer keyboard.

