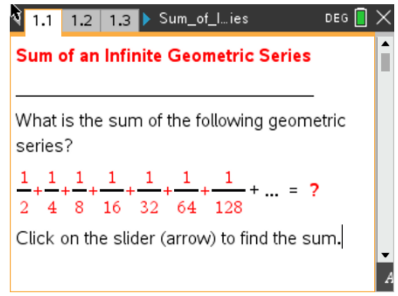
Summing Up Geometric Series

Student Activity

Name ______________

Class ______________

In this activity, you will explore the concept of finding the sum of an infinite geometric series. Reviewing the concepts of when you can find the sum of an infinite geometric series will be the first task, discussing with your classmates not only what the characteristics of a geometric sequence are, but also the key characteristic that allows you to add every term of the infinite sequence and still get a non-infinite sum.



Let us review the characteristics of a geometric sequence. A **geometric sequence** is a sequence of terms where the ratio of every two consecutive terms is constant. The constant ratio is referred to as the **common ratio** or ***r***. To find subsequent terms of a geometric sequence, multiply a term by ***r***. The nth term, u_n, formula for a geometric sequence is $\boldsymbol{u_n = u_1 \cdot r^{n-1}}$, where u_1 is the first term, and *r* is the common ratio.

Problem 1 – Geometric Sequence Practice

1. Find the next three terms of each infinite geometric series.

   (a) $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots$

   (b) $2+\frac{3}{2}+\frac{9}{8}+\ldots$

2. Discuss with a classmate how you would find the next three terms of each series. Explain your results.

3. Sigma notation is used at times to express a series. The symbol for sigma, Σ, actually means the sum of. Using the nth term formula from above and the sigma notation, write an expression in terms of *n* that describes each of the series from number 1.

4. Discuss with a classmate how you would find the sum of each series in number 1. Explain your results.

Problem 2 – Finding the Sum of a Geometric Series

There are two types of geometric series. There is the partial sum or finite series and then there is the infinite series.

1. Discuss with a classmate the formula to find a partial sum of a geometric series. Explain what you would need to use the formula and if there are any restrictions.

2. Explain how you could use sigma notation to find the partial sum of a geometric series as well. Explain what you would need to use sigma.

3. Discuss with a classmate the formula to find an infinite sum of a geometric series. Explain what you would need to use the formula and if there are any restrictions.

4. Explain how you could use sigma notation to find the infinite sum of a geometric series as well. Explain what you would need to use sigma.

5. Given the geometric sequence 1, -4, 16, -64, …, find the partial sum of the first 9 terms.

6. Given the geometric sequence 9, 3, 1, $\frac{1}{3}$, …, find the infinite sum.

7. Write the sequences in questions 5 and 6 in sigma notation. Explain if you can use your handheld to verify your answers using sigma notation.

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Problem 3 – Visualizing an Infinite Geometric Series

Open the tns file Sum_of_an_Infinite_Geometric_Series.tns. In this file, you will be shown how a unit square can be divided into an infinite number of pieces. **Move to page 1.2.**

1. Select ▲ to see a rectangle with a shaded area of $\frac{1}{2}$ unit^2. The length of a side of the original square is 1 unit. Write down the dimensions of the rectangle.

2. Select ▲ until the shaded area increases to $\frac{7}{8}$. Write down the dimensions of the three rectangles whose sum is $\frac{7}{8}$.

3. Discuss with a classmate what you expect the area of the next rectangle to be added to the sum would be. (Express your area in both fractional and decimal forms.) Explain how you arrived at your conclusion.

4. Continue selecting ▲ until you can't press it any more. If you could select ▲ again, write down what you think the area of the next rectangle would be. Find what the total sum of the areas would be. (Express your answers in both fractional and decimal forms.)

5. If you could continue selecting ▲ an infinite number of times, and the whole region were shaded, describe what the total shaded area would be. (Express your answers in both fractional and decimal forms.)

6. Write an expression for the sum of the areas of the infinite number of rectangles formed. Find the value of this sum. Explain how you found this sum.

7. Express your answer from Question 6 in sigma notation.

8. Instead of halving the side of the square, suppose that we doubled its size and continued to double a side of each subsequent square formed.
   a. Express the sum of the areas of these squares as an infinite sum.

   b. Describe what happens to this sum as the number of squares increases. Explain your answer.

9. Instead of halving the length or width of each of the rectangles, suppose that we multiplied the rectangle's length or width by $\frac{1}{3}$, giving us the series $\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\frac{1}{81}+\ldots$

   State if you think that the sum of the series would be finite or infinite. Explain.

10. Give an example of an infinite geometric series that you think would have a finite sum and an example of one that you think would not have a finite sum. Explain your reasoning.

11. Based on the information above, describe what conjecture that must be true about the ratio of the consecutive terms of an infinite geometric series for the series to have a finite sum.

Move to page 1.3.

12. Find the values of the ratio *r* where an infinite geometric series appears to have a finite sum.

<u>Further IB Application</u>

A local coffee shop had an amazing first year after it opened, earning $40,000 of profit. Unfortunately, the profits have been decreasing by 10% each year after the first. Assuming that this trend continues, find the total profits the shop hopes to earn over the course of its lifetime.

