Center of Mass

MATH NSPIRED

Math Objectives

- Students will identify and interpret the mean geometrically as the location such that the sum of the distances on either side of the mean is the same.
- Students will recognize that the center of balance for a set of univariate data is the weighted mean.
- Students will model with mathematics and use appropriate tools strategically (CCSS Mathematical Practices).

Vocabulary

- centroid
- center of balance
- fulcrum
- mean
- median
- maximum
- minimum
- range
- weighted mean

About the Lesson

- This lesson involves locating the point along a 12-inch ruler that will balance five quarters taped to various locations.
- As a result, students will:
  - Drag the fulcrum on the ruler and observe that the point that balances the arrangement will be the position in which the sum of the moments, or the product of the Force (F) and the distance (d) from the fulcrum, is equal on each side of the fulcrum.
  - Identify the center of balance (or the center of mass) as the point where the sums of the distances on each side of the fulcrum are equal.
  - Discover the physical balance point is associated with the arithmetic mean of the numerical locations of the quarters

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- Send the .tns file to students.
- Use Quick Poll to assess students' understanding.

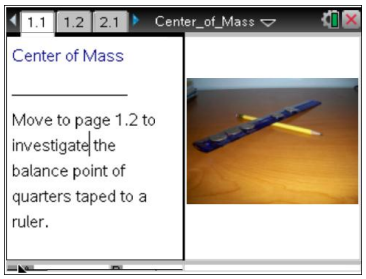
Prerequisite knowledge

Students should have prior experience:

- Calculating the mean and median when given a set of data.
- Identifying and interpreting data represented on a dot plot.

Activity Materials

- Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software



Tech Tips:

- This activity includes screen captures taken from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:

Student Activity

- Center_of_Mass_Student.pdf
- Center_of_Mass_Student.doc

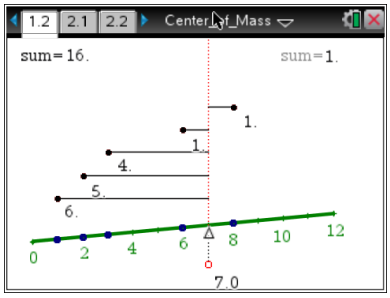
TI-Nspire document

- Center_of_Mass.tns

Discussion Points and Possible Answers

Teacher Tip: Consider beginning this investigation by actually modeling the scenario described. Use a lightweight 12-inch ruler. Tape five quarters to the ruler at the locations designated in the problem. Place a pencil under and perpendicular to the ruler so that it acts as a support, or fulcrum. Students can try to move the pencil to a position that balances the quarters. The geometric, or physical, balance point will be approximately the arithmetic mean of the numerical locations of the quarters. Although other factors contribute to the actual position of the balance point (for example, the varying weights of the quarters), the mathematical goal of this investigation is to discover the relationship between the mean and the balance point.

Move to page 1.2.



Five quarters are taped at locations along a lightweight 12-inch ruler. A pencil is placed under the ruler that is also perpendicular to the ruler. The pencil is carefully moved along the bottom side of the ruler to find a location that balances the arrangement of the quarters on the ruler

1. On the graphic representation on Page 1.2, the blue points represent the location of the 5 quarters along a 12-inch ruler. Identify the location of the quarters.

   Answers:
   a. Quarter 1 is at 1 inch.
   b. Quarter 2 is at 2 inches.
   c. Quarter 3 is at 3 inches.
   d. Quarter 4 is at 6 inches.
   e. Quarter 5 is at 8 inches.

2. The position of a ***fulcrum*** is represented by the symbol "Δ." The pencil serves as the fulcrum in this problem because it supports the ruler and the quarters. Note on Page 1.2 that a vertical line is drawn to represent the location of the fulcrum.
   a. What is the location of the fulcrum along the bottom of the ruler?

Answer: The fulcrum is at 7 inches.

b. Does this location of the fulcrum balance the quarters? Explain your answer.

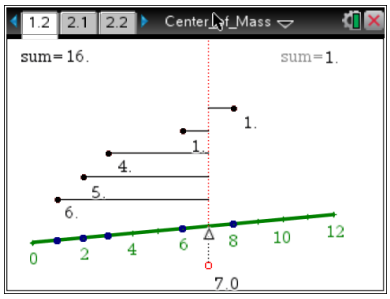
Sample Answers: This location does not balance the arrangement of quarters. The left sum and the right sum are different, and the ruler is clearly unbalanced.

c. How do you think the ruler would look if the pencil balanced the quarters?

Sample Answers: Students could describe an arrangement or provide a sketch in which the ruler would be level, or horizontal. Students should identify a position to the left of 7 as the balance point as they recognize that to balance the quarters the fulcrum needs to be moved towards the three quarters that weigh down the ruler.

How is balance achieved? The first step is to consider a physics concept, a ***moment arm***. The force of gravity pulls each quarter downward. The pull of the quarters on one side of the fulcrum must equal the pull on the other side for the position of the fulcrum to balance the quarters. The pull that a quarter provides, called the "torque" or the "moment" *(M)*, is a product of the force (*F*) or weight of the quarter times the distance (*d*) of the quarter from the fulcrum. This description is summarized mathematically as: $M = F \times d$.

3. Let the force or weight of a quarter be represented by *F*. Consider the quarter located at 3 inches on the ruler with the fulcrum located at 7 inches. The moment that this quarter exerts would be represented as $4 \times F$, or $4F$ since the quarter is a distance of 4 units from the fulcrum.



   a. What is the sum of the moments exerted by the quarters to the right of the fulcrum?

Sample Answers: F represents the quarter located at position 8 on the ruler as 1 unit to the right of the fulcrum.

Teacher Tip: The unit representing moment in a physics class is a Newton meter or foot-pounds. For this lesson, however, students can simply refer to this as a "unit."

b. What is the sum of the moments exerted by the quarters to the left of the fulcrum?

Sample Answers: 6F units + 5F units + 4F units + 1F unit = 16F units

c. Explain why the ruler would or would not balance if the fulcrum were located along the bottom of the ruler at 7 inches.

Sample Answers: The location of the fulcrum at position 7 would not balance the ruler as the 4 quarters to the left result in a greater moment. As shown on page 1.2, the ruler is weighted heavily to the left.

Teacher Tip: When comparing the moments, the force (or weight) of each quarter is considered equal; therefore, the quarters provide an equal unit of force. As a result, locating the balance point only requires students to focus on the distances on each side of the fulcrum. The following questions direct students to observe the distances from fulcrum. Students discover that the mean of the data is also the value in which the sums of the distances on each side of the mean are equal; therefore, the total moments on each side of the fulcrum would be equal.

4. a. What is the sum of the ***distances*** of the quarters located on each side of the fulcrum?

Sample Answers: The sum of the distances on the right is 1 inch as there is only 1 quarter to the right of the fulcrum. The sum of the distances on the left is 16 inches.

b. Make a conjecture about what you think happens to the sum of the distances as the fulcrum is moved left to a position that will balance the quarters. Make a sketch of the ruler and fulcrum to illustrate your conjecture.

Sample Answers: The balance point would be found by moving the pencil to the left, or towards the pull of the quarters on that side of the ruler. As students move the pencil, the sum of the distances on the left decrease, while the sum of the distances on the right increase. A picture of this arrangement would indicate that the ruler is horizontal.

5. To test your conjecture in 4b, grab the point on the bottom of the ruler representing the position of the fulcrum and move it to the left.
   a. What happens to the sum of the distances on the right and the sum of the distances on the left as you move the fulcrum to the left?

      Sample Answer: The sum of the distances increases on the right and decreases on the left as you move the fulcrum to the left.

   b. Move the fulcrum until it appears balanced. Describe the picture in terms of the ruler.

      Sample Answer: The ruler is horizontal when the fulcrum is located at 4 inches.

   c. Consider the position of the fulcrum when the ruler is horizontal. What is the sum of the distances on each side of the fulcrum?

      Answer: The sum of the distances on both the right and left is 6 inches.

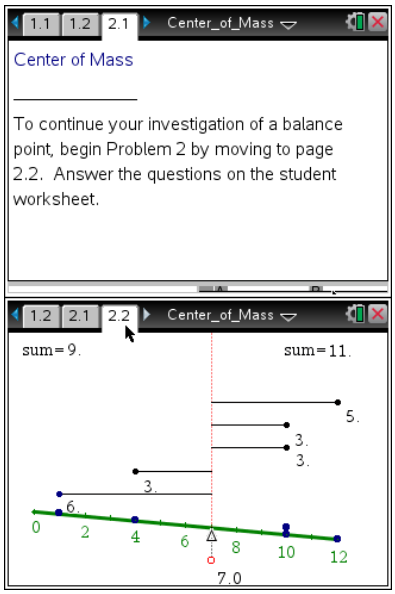
   d. Explain how the position of the fulcrum that balances the quarters confirms or does not confirm you conjecture in question 4b.

      Sample Answers: Students would discover this point balances the quarters and is the point where the sums of the distances on each side of the fulcrum are equal.

6. Many different summary statistics can characterize a data set. Some common summary statistics are the minimum, maximum, mean, median, and range. Each of these statistics is valuable and measures a specific characteristic of the data set. Determine the minimum, maximum, mean, median, and range of the locations of the quarters on the ruler. Do any of these summaries support your conjecture about the balance point? Explain your answer.

   Sample Answers: The minimum is 1 inch, the maximum is 8 inches, the mean is 4 inches, the median is 3 inches, and the range is 7 inches. The mean of the distances is the location of the fulcrum that balances the ruler.

Move to pages 2.1 and 2.2.



7. The five quarters on the ruler are rearranged on Page 2.2. Identify the location of each quarter.

   Answers:
   a. Quarter 1 is at 1 inch.
   b. Quarter 2 is at 4 inches.
   c. Quarter 3 is at 10 inches.
   d. Quarter 4 is at 10 inches.
   e. Quarter 5 is at 12 inches.

8. Move the fulcrum to the 7-inch mark.
   a. Let *X* equal the distance of the quarters from the fulcrum to the 10-inch mark. The sum of the distances to the right of the fulcrum would be equal to the following: $2X + 5$. Determine the value of the *X*. Why is the value represented by *X* multiplied by 2?

      Sample Answer: The *X* would be equal to 3 inches. It is multiplied by 2, since there are two quarters located at the location 10 inches.

   b. What is the sum of the distances to the right of the fulcrum?

      Answer: The sum of the distances to the right is 2(3) + 5, or 11 inches.

   c. What is the sum of the distances to the left of the fulcrum?

      Answer: The sum of the distances to the left is 6 + 3, or 9 inches.

   d. Calculate the sum of the distances if you were to tape another quarter at position 10.

      Sample Answers: You would multiply by 3 the value of the distance from the 7 to 10 inches. Therefore, the sum of the distances to the right would be 3(3) + 5, or 14 inches. The sum of the distances to the left would remain at 9 inches.

9. Move the fulcrum to a point that will approximately balance the quarters.
   a. How do you know you have found the balance point? What is the estimate of this position?

      Sample Answers: Move the fulcrum to the right. When the ruler is horizontal, the quarters and ruler would balance. This position is approximately 7.4 inches.

   b. Is the location of the balance point the same as the location suggested by the numerical calculations? Explain.

      Sample Answers: The position 7.4 inches equalizes the sum of the distances on both sides of the fulcrum. 7.4 is also the mean of the values 1, 4, 10, 10, and 12.
      Note: Mean = $\frac{1+4+2(10)+12}{5} = 7.4$

10. a. What other factors do you think might also affect the location of the balance point?

      Sample Answers: The weight of the ruler, along with the weight of the tape used to attach the quarters to the ruler, are part of the physical model that determines the balance point. See the Teacher Note provided after the Wrap Up section to help students begin to understand how the force due to gravity is also acting on the ruler.

   b. Why might the balance point be called the *weighted mean* in this second problem?

      Sample Answers: The stacked quarters at position 10 results in that position having more weight. Any position in which there are more quarters attached will result in a more weight exerted at that position. The mean of the distances, therefore, will be found by multiplying those positions by the number of units (or quarters) taped to those positions.

> **Teacher Tip:** Consider giving students an extension of Problem 2. Suppose eight quarters are taped to various locations along the ruler with two quarters stacked on top of each other at one location and three quarters on top of each other at another location. Ask students to find the balance point and then share their "rulers" and their answers to the question. This extension will further develop the concept of weighted mean and an understanding of the objectives of this lesson.

Wrap Up

Upon completion of this investigation, students should be able to understand:

- The mean of a set of data is the value such that the sum of the distances from the mean to the individual values is the same on both sides of the mean.
- The mean of the physical model described in this investigation is the balance point.

Teacher Note: The mean of the locations of the quarters represents the point along the ruler that is the center of mass, or the balance point, provided the ruler is "weightless." This point is also called the **centroid**. If you develop the physical model with students, you will notice that the mean will not exactly represent the balance point. The force of gravity also pulls down the ruler. This force results in adding a moment to the investigation that also affects the actual location of the balance point. If the ruler is thin, the force due to gravity on the quarters will be the primary force. This investigation assumes that these other factors are minimal and, therefore, the mean of the positions of the quarters will provide a good estimate to the balance point. The physical model will, of course, be affected by these other factors.

Assessment

1. Suppose a quarter is 4 units to the right of the fulcrum, which is located at 6.

a. What is the moment of the quarter?
Answer: 4(1) or 4

b. If there were two quarters to the left of the fulcrum, where would they have to be placed to balance the fulcrum?
Answer: Both of the quarters could be at 4, 2 units from the fulcrum or one quarter would be 5, 1 unit away and one at 3, 3 units away.

c. Find the weighted average of the quarters and explain how it relates to the fulcrum.
Answer: The weighted average of the quarters would be 6 by averaging either 4, 4,10 or 3, 5, 10. It is where the fulcrum would have to be placed to balance the quarters.

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A *Quick Poll* can be given at the conclusion of the lesson. You can save the results and show a Class Analysis at the start of the next class to discuss possible misunderstandings students might have.