Simple Harmonic Motion

ID: 10038

Time required
45 minutes

Activity Overview

Students are introduced to the important topic of simple harmonic motion with the example of the motion of a child on a swing. Using multiple representations to support understanding, students derive the equations of motion. The activity begins with the trigonometric function between time and displacement and differentiates up to the form for acceleration. Then, it concludes with integration from acceleration back to displacement. This activity provides an introduction to differential equations.

Topic: Applications of Integration

- *Solve the differential equation for simple harmonic motion and graph its solution to explore its extrema.*

Teacher Preparation and Notes

- *This activity can serve to consolidate earlier work on differentiation and integration. It offers a suitable introduction to differential equations.*
- *Begin by reviewing the method of differentiation of trigonometric functions and methods of integration of the standard function forms.*
- *This activity is intended to be teacher-led.*
- *This activity requires the use of TI-Nspire CAS technology.*
- *Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.*
- ***To download the student and solution TI-Nspire documents (.tns files) and student worksheet, go to education.ti.com/exchange and enter "10038" in the keyword search box.***

Associated Materials

- *SimpleHarmonicMotion_Student.doc*
- *SimpleHarmonicMotion_Soln.tns*
- *SimpleHarmonicMotion.tns*

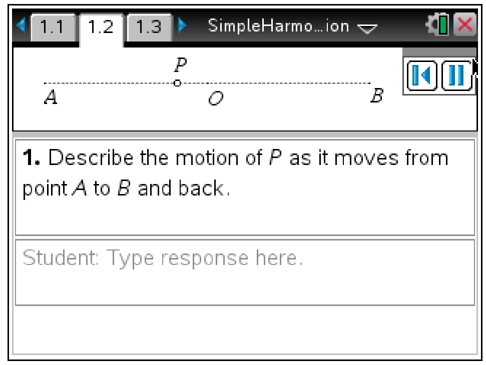
Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *Differential Equations (TI-Nspire CAS technology) — 8998*
- *Damped and Driven Harmonic Motion (TI-Nspire CAS technology) — 9523*
- *Gettin' the Swing (TI-Nspire technology) — 11689*

Problem 1 – Motion of a Swing

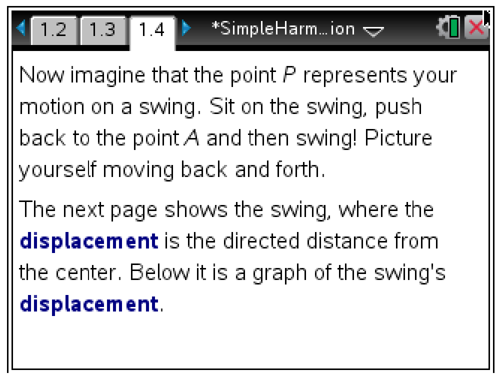
Begin with discussion and review of both the differentiation and integration of standard forms and of the trigonometric functions in particular. Ensure that students are comfortable with these forms and then challenge them to apply what they know about these processes to real-world situations, with a particular focus upon rates of change.



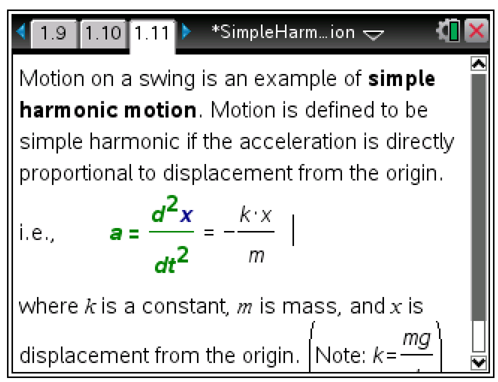
TI-Nspire™ Navigator™ Opportunity: Class *Capture*

See Note 1 at the end of this lesson.

Motion on a swing should be something to which all students can relate. It offers a suitable context for a closer examination of rate of change of displacement leading to velocity, and rate of change of velocity leading to acceleration.



Particular care should be taken to build understanding of the concept of acceleration—while students will readily understand displacement and velocity, acceleration is best introduced in terms of an applied force (being careful to distinguish between them!). Once established, this forms the basis for deriving the forms for simple harmonic motion, beginning with the standard relationship between time and displacement, leading to velocity, and finally to acceleration. Substituting back leads readily to the defining equation in terms of acceleration and displacement.

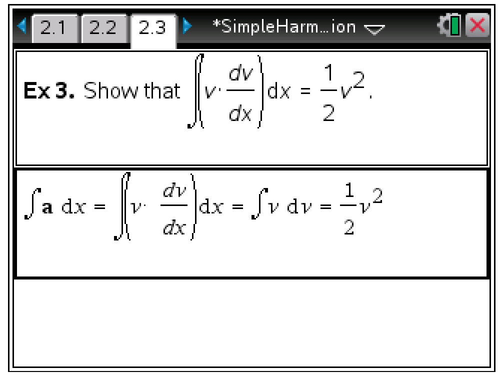


TI-Nspire™ Navigator™ Opportunity: *Quick Poll*

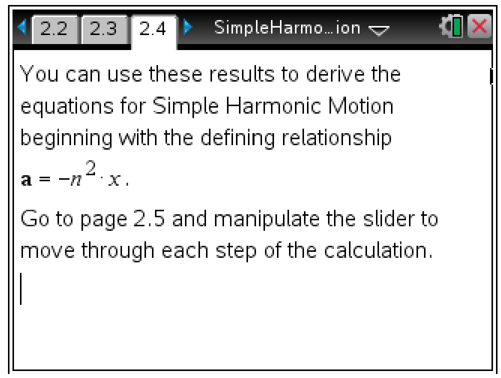
See Note 2 at the end of this lesson.

Problem 2 – Extension

The second part of this activity offers a series of extensions, beginning with the challenge for students to describe other examples of simple harmonic motion. They should then be introduced to the variety of forms for acceleration, and attempt to justify these from their knowledge of rates of change.



What follows supports students in deriving the simple harmonic forms by integrating from the acceleration equation. This involves some substitution of critical values drawn from the physical example of the child on the swing and, finally, integration of inverse trigonometric function forms. When deriving this form, students should be encouraged to discuss the use of the sine or the cosine form – each appropriate depending upon the physical conditions. This should lead to consideration of the **phase shift** as related to the starting point of the motion.



Student Solutions

1. The motion is cyclic—initially negative (moving towards the origin, *O*) then positive. Velocity reaches a maximum value in the middle and is zero at end-points *A* and *B*.
2. Let *OA* = *OB* = *A* (**amplitude**) and assume the particle takes *T* seconds to complete one cycle. *T* is called the period. The position of the particle at any time *t* is $x = A \cdot \sin\left(\frac{2\pi}{T} \cdot t\right)$.

   The frequency, *f*, of the particle is defined as $f = \frac{1}{T}$. Let *n* be $2\pi \cdot \frac{1}{T} = 2\pi \cdot f = n$. So, the equation of motion changes to $x = A \cdot \sin(n \cdot t)$.
3. You move fastest as you pass through the center of the swing's path, and you stop (briefly) at each end of the path—just as our point *P*!
4. Acceleration will be greatest when you are furthest from the ground—at each end of the path. It will be least when you are at your lowest point, in the middle.
5. If the origin of motion is taken to be the centre of the swing's path (where acceleration/force is least) then the further you move away from that position, the greater the acceleration/force acting upon you trying to return you to that position.

6. If $x = A \cdot \sin(n \cdot t)$, then $v = \dfrac{dx}{dt} = A \cdot n \cdot \cos(n \cdot t)$.

7. $a = \dfrac{d^2}{dt^2}(x) = \dfrac{d}{dt}\left(A \cdot n \cdot \cos(n \cdot t)\right) = -A \cdot n^2 \cdot \sin(n \cdot t)$

8. Given $x = A\sin(nt)$ and $a = -An^2\sin(nt)$ then:
$a = -n^2 A\sin(nt)$
$a = -n^2 x$

9. The force/acceleration acting upon a child on a swing acts in a negative direction to the motion, proportional to the square of the period and the displacement from the origin—in other words, the further you are from the “rest position” the more force there is from gravity to return you there!

10. At point *O*, displacement and acceleration are zero, while velocity has a maximum value. At the end points, velocity is zero, while displacement and acceleration attain their maximum values.

Ex 1: Most musical instruments (e.g., guitar string when plucked, reed in a wind instrument - hence the name simple harmonic motion). The tides and even a cork or object moving up and down with the tides.

Ex 2: $a = \dfrac{d^2}{dt^2}(x) = \dfrac{d}{dt}\dfrac{dx}{dt} = \dfrac{dv}{dt}$

$a = \dfrac{dv}{dx} \cdot \dfrac{dx}{dt} = v \cdot \dfrac{dv}{dx}$

Ex 3: $\int a\,dx = \int\left(v \cdot \dfrac{dv}{dx}\right)dx = \int (v)dv = \dfrac{1}{2}v^2$

Ex 4: Depending on where the motion began: if $x = 0$ when $t = 0$, then sine is most appropriate. However, if the motion began from one of the end points (someone pulling the swing back before releasing) then cosine would be the better choice!

TI-Nspire™ Navigator™ Opportunities

Note 1
Problem 1, *Class Capture*
Use Class Capture to verify that students are able to run the animations and use the images to draw out a class discussion.

Note 2
Problem 1, *Quick Poll*
Use Quick Poll to assess student understanding. The worksheet questions can be used as a guide for possible questions to ask.