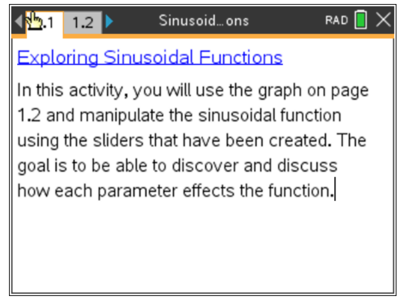


Math Objectives

- Students will approximate the zeros, minima, maxima, and period of sinusoidal functions by using the **Transformation Graphing** app.
- Students will approximate the amplitude, frequency, and phase shift of sinusoidal functions by graphing.
- Given the equation of a sinusoidal function, students will state its range, amplitude, frequency, period, and phase shift.
- Students will describe how the graph of sinusoidal functions, $y = f(x)$, changes under transformations.

Vocabulary

- amplitude
- period
- frequency
- parameter
- phase shift

About the Lesson

- This lesson involves the sinusoidal of the form

$$f(x) = A \cdot \sin\big(B(X + C)\big) + D.$$

- As a result, students will:
- Manipulate parameters and observe the effect on the graph of the corresponding sinusoidal function.
- Make general statements about the effect of each parameter on the graph of the sinusoidal function.

TI-Nspire™ Navigator™

- Transfer a File.
- Use Class Capture to examine patterns that emerge.
- Use Live Presenter to demonstrate.
- Use Teacher Edition computer software to review student documents.
- Use Quick Poll to assess students' understanding

Activity Materials

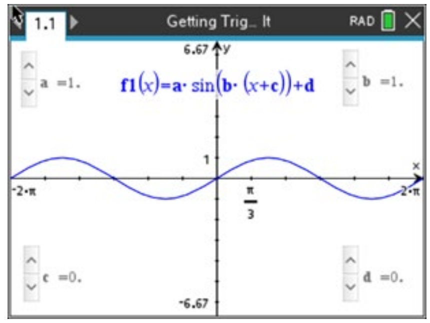
Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software

Tech Tips:

- This activity includes screen captures taken from the TI-Nspire CX II handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:
Sinusoidal Functions_Nspire_Student.pdf
Sinusoidal Functions_Nspire_Student.doc
Sinusoidal Functions.tns

In this activity, students systematically explore the effect of the coefficients on the graphs of sinusoidal functions. Terminology describing the graph—amplitude, period, frequency, phase shift, midline, and vertical offset—is introduced, then reinforced as the student calculates these values directly from the graph using the graphing calculator and sliders.



Discussion Points and Possible Answers

> **Tech Tip:** To change the parameters throughout this activity using sliders on the handheld, you will be pressing the up/down arrows for each individual parameter. Try encouraging your students to use both positive and negative values for each parameter. You may choose to not download the file and have the students create the sliders. Make sure you show the students how to change the settings for each slider.

The parameters a, b, c and d will each affect your sinusoidal function in different ways. You will be using sliders on the handheld to change the value of a parameter by pressing the arrows of each individual slider and manipulating them. You will download the file ***Sinusoidal Functions.tns.*** At the end of this activity, you will have a much better understanding of the role of each parameter and how they affect a sinusoidal function.

Problem 1 – A general trigonometric function

Once the file has been downloaded, go to page 1.2 where the sinusoidal function below has been entered and sliders created.

$\boldsymbol{f1(x) = a \cdot \sin(b(x+c)) + d}$.

(a) Complete the table.

a	b	c	d	zero1	zero2	min	max
1	1	0	0	**0**	$\approx \mathbf{3.14}$	**-1**	**1**
4	½	3	1	$\approx \mathbf{3.636}$	$\approx \mathbf{9.091}$	**-3**	**5**

(b) With a classmate, write down the differences you notice between the graph created by row one and the graph created by row two.

Sample Answers: The zeros have shifted horizontally, min decreased and the max increased and the min and max seemed to get farther apart.

Problem 2 – The effect of the coefficients a, b, c, and d

Examining a

(a) Set *b* = 1 and *c* = *d* = 0 and change the value of *a*. Try 4 different values of *a* (even negative values) and fill in the table below. **(**a values may vary**)**

a	b	c	d	zero1	zero2	min	max
-2	1	0	0	**0**	$\approx \mathbf{3.14}$	**-2**	**2**
-1	1	0	0	**0**	$\approx \mathbf{3.14}$	**-1**	**1**
0.5	1	0	0	**0**	$\approx \mathbf{3.14}$	**-0.5**	**0.5**
4	1	0	0	**0**	$\approx \mathbf{3.14}$	**-4**	**4**

(b) How did the appearance of the graph change?

Sample Answer: The graph stretches vertically.

(c) Which graph features changed? Which did not change?

Sample Answers: The mins and maxs changed, but the zeros did not.

(d) Write equations to describe the relationship between *a* and the features that did change.

Sample Answers: $f(x) = -2\sin x \;\; or\; f(x) = .5\sin x\,, etc.$

The value of $|a|$ is the **amplitude**. It is equal to half of the difference between its maximum and minimum values.

(e) Calculate the amplitude from the minimum and maximum values in the table above.

Sample Answers: $|-2| = 2 \quad and \quad 2-(-2) = 4\cdot\frac{1}{2} = 2$

(f) Compare the results to the values of *a*. What do you notice?

Sample Answers: They are all equal to the absolute value of the a that was chosen.

Examining b

(a) Set *a* = 1 and *c* = *d* = 0 and change the value of *b*. Try 4 different values of *b* (even negative values) and fill in the table below. **(**b values may vary**)**

a	*b*	*c*	*d*	zero1	zero2	min	max
1	**-4**	0	0	**0**	**0.795**	**-1**	**1**
1	**0.5**	0	0	**0**	**6.307**	**-1**	**1**
1	**3**	0	0	**0**	**1.080**	**-1**	**1**
1	**5**	0	0	**0**	**0.625**	**-1**	**1**

(b) How did the appearance of the graph change?

Sample Answers: The number of curves/cycles/periods seems to either increase or decrease.

(c) Which graph features changed? Which did not change?

Sample Answers: The distance from peak to peak or trough to trough changes, how many cycles changes, and almost all zeros change. The mins and maxs did not change and it still passes through the origin.

(d) Describe the relationship between *b* and the features that did change.

Sample Answers: If the length of one period (cycle) of the parent function of a sinusoidal function is 2π, there seems to be a relationship between the b value and 2π. It also seems like the distance between the zeros in the table above is about half the distance in the relationship between b and 2π.

(e) What **two** features of the sinusoidal function can the parameter *b* help you find? Define them both.

Sample Answers: Period and Frequency. Period is the length of one complete cycle $\left(\frac{2\pi}{|b|}\right)$. Frequency is the number of cycles per 2π $\left(\frac{|b|}{2\pi}\right)$.

Examining c

(a) Set *a* = *b* = 1 and *d* = 0 and change the value of *c*. Try 4 different values of *c* (both positive and negative values) and fill in the table below.

a	*b*	*c*	*d*	zero1	zero2	min	max
1	1	**-2**	0	**-1.193**	**1.989**	**-1**	**1**
1	1	$-\frac{\pi}{3}$	0	**-2.140**	**1.042**	**-1**	**1**
1	1	**1**	0	**-1.004**	**2.178**	**-1**	**1**
1	1	$\frac{\pi}{4}$	0	**-0.720**	**2.348**	**-1**	**1**

(b) How did the appearance of the graph change and what do we call that change?

Sample Answers: The graph seems to shift horizontally (left or right). A phase shift.

(c) Which graph feature changed? Which did not change?

Sample Answers: The zeros changed. The mins and maxs did not change.

(d) What is the effect of an increasing sequence of values for *c* on the graph?

Sample Answers: The function will vertically shift to the left.

(e) What is the effect of a decreasing sequence of values for *c* on the graph?

Sample Answers: The sinusoidal function will vertically shift to the right.

Examining d

(a) Set $a = b = 1$ and $c = 0$ and change the value of *d*. Try 4 different values of *d* (both positive and negative values) and fill in the table below.

a	*b*	*c*	*d*	zero1	zero2	min	max
1	1	0	-3	**None**	**None**	**-4**	**-2**
1	1	0	-0.5	**0.568**	**2.614**	**-1.5**	**0.5**
1	1	0	1	**-1.515**	**4.659**	**0**	**2**
1	1	0	4	**None**	**None**	**3**	**5**

(b) How did the appearance of the graph change?

Sample Answers: The function seems to shift vertically up or down.

(c) Try an increasing sequence of values for *d* such as 0, 1, 2, 3, 4…
What is the effect on the graph?

Sample Answers: The function is shifted up d units.

(d) Try a decreasing sequence of values for *d* such as 0, –1, –2, –3, –4…
What is the effect on the graph?

Sample Answers: The function is shifted down d units.

(e) Describe the effect of the value of *d* on the graph. How does changing *d* change the graph features?

Sample Answers: The effect of d is a vertical translation of the sinusoidal graph. The zeros change as the graph moves up or down and the minimums or maximums increase or decrease depending on the value of d.

Problem 3 – A closer look at amplitude, period, and frequency

In $f1(x)$, enter the general cosine function, $\boldsymbol{f1(x) = a \cdot \cos(b(x+c)) + d}$

amplitude: half of the vertical distance from minimum value to maximum value

period: horizontal distance from one peak (maximum point) to the next or one minimum point to the next

frequency: number of cycles per 2π interval

- Write a formula to find the frequency *f* given the period *p*.

  Sample Answers: $f = \frac{1}{p}$

- Use the formula to complete the table below.

a	*b*	*c*	*d*	max point	min point	next max point	amplitude	period	frequency
1	1	0	0	(0, 1)	(3.14, –1)	(6.28, 1)	½*(1 – (–1)) 1	6.28 – 0 6.28 2π	$\frac{1}{2\pi}$
-3	1	0	0	**(3.14, 3)**	**(0, -3)**	**(9.42, 3)**	$\frac{1}{2}(3-(-3))$ **3**	$\mathbf{9.42-3.14}$ $\mathbf{6.28}$	$\frac{1}{2\pi}$
2	1	0	0	**(0, 2)**	**(3.14, -2)**	**(6.28, 2)**	$\frac{1}{2}(2-(-2))$ **2**	$\mathbf{6.28-0}$ $\mathbf{6.28}$	$\frac{1}{2\pi}$
1	**-1**	0	0	**(0, 1)**	**(3.14, -1)**	**(6.28, 1)**	$\frac{1}{2}(1-(-1))$ **1**	$\mathbf{6.28-0}$ $\mathbf{6.28}$	$\frac{1}{2\pi}$
1	$\frac{1}{2}$	0	0	**(0, 1)**	**(6.28, -1)**	**(12.6, 1)**	$\frac{1}{2}(1-(-1))$ **1**	$\mathbf{12.6-0}$ $\mathbf{12.6}$ $\mathbf{4\pi}$	$\frac{1}{4\pi}$
1	1	$-\frac{\pi}{2}$	0	**(1.57, 1)**	**(4.71, -1)**	**(7.85, 1)**	$\frac{1}{2}(1-(-1))$ **1**	$\mathbf{7.85-1.57}$ $\mathbf{6.28}$	$\frac{1}{2\pi}$
1	1	$\frac{\pi}{6}$	0	**(-.52, 1)**	**(2.62, -1)**	**(5.76, 1)**	$\frac{1}{2}(1-(-1))$ **1**	$\mathbf{5.76}$ $-(-.52)$ $\mathbf{6.28}$	$\frac{1}{2\pi}$
1	1	0	**-6**	**(0, -5)**	**(3.14, -7)**	**(6.28, -5)**	$\frac{1}{2}(1-(-1))$ **1**	$\mathbf{6.28-0}$ $\mathbf{6.28}$	$\frac{1}{2\pi}$
1	1	0	**4**	**(0, 5)**	**(3.14, 3)**	**(6.28, 5)**	$\frac{1}{2}(1-(-1))$ **1**	$\mathbf{6.28-0}$ $\mathbf{6.28}$	$\frac{1}{2\pi}$

- Based on the results in the table, discuss with a classmate and record each relationship:

  a and amplitude *b* and the frequency *b* and the period

Sample Answers:

$|A| = amplitude$ $\frac{|B|}{2\pi} = frequncey$ $\frac{2\pi}{|B|} = Period$

Teacher Note: Many of the decimals in the tables may be slightly different from student to student depending on how they use the technology to find them. Please remember that they are approximates. If you would like more uniformity, then time should be spent discussing a certain method to approximate the values.