

Transformations of Functions

Student Activity

7 8 9 10 11 12



TI-84Plus™ CE

Investigation

Student

40 min

Introduction

Functions can be reflected, dilated or translated, collectively these are called transformations. The purpose of this activity is to focus on the transformations. Data has been collected using a variety of sensors to measure the following:

- Motion of a bouncing ball
- Motion of a pendulum
- Discharge of a capacitor

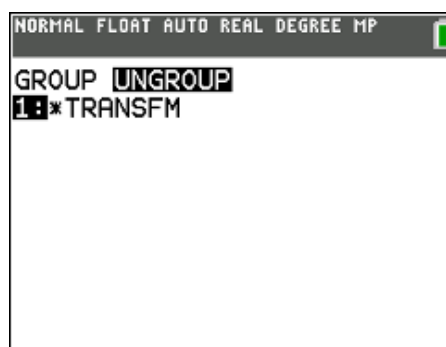
For each of these sets of data, an appropriate function has been defined in Y_1 . Transformations to this function can be applied using Y_2 .

The Group file: TRANSFM contains the data sets and three programs, one for each data set to help set up the calculator.

To open the Group file from the home screen press:

$\boxed{2nd}$ $\boxed{[mem]}$ – Select Option 8 [Group] then Ungroup

The group file contains lists: $L_1 \dots L_6$ and some program files. When prompted at the “Duplicate Name” menu, select “Overwrite All”



Calculator Tip!

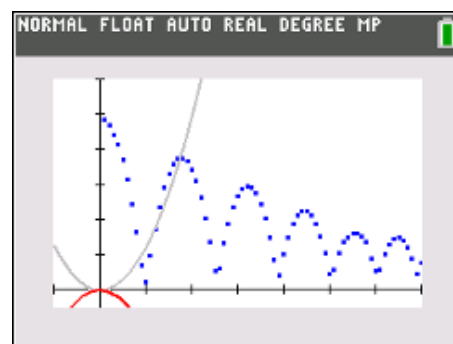


The investigation jumps in and out of the graph screen. To speed up the graphing process the XRES (x resolution) has been changed to 3, this means graphs may appear slightly distorted as fewer points are used to graph each function. When you want to see the graph in more detail change the XRES in the window settings back to the default value of 1.

Bouncing Ball

Run the program “Prob1”. This will select the data for the ball bounce problem, set up a scatter plot and enter the basic function in Y_1 .

In Y_2 enter the equation: $-Y_1$.



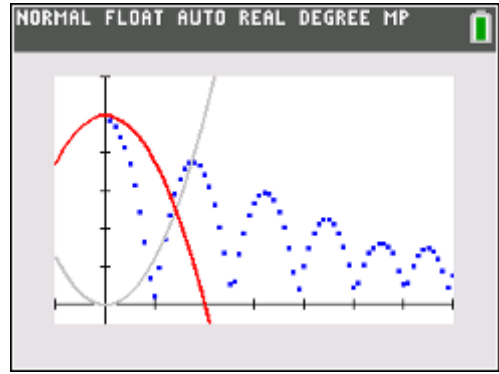
Question 1

How does this first transformation affect the graph?

The next step is to provide explore another transformation of the function. Edit the function defined by Y_2 as $-Y_1 + k$ by trying different values of k with an attempt to model the first 'drop' of the bouncing ball.

Note:

It is not essential at this stage to get the ideal value of k , something relatively close is sufficient as it can be edited again later.



Question 2

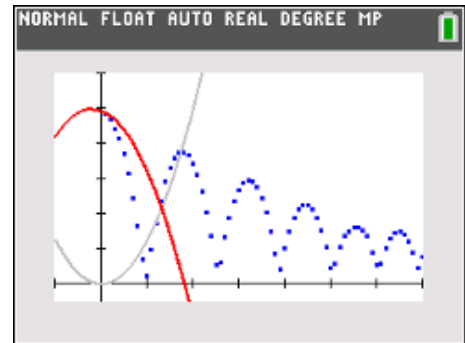
How does this transformation affect the graph?

The next step is to provide explore another transformation of the function. Edit the function again, keeping all previous transformation:

$-Y_1(x - h) + k$ by trying different values of h .

Note:

Once again, it is not essential at this stage to get the ideal value of h , something relatively close is sufficient as it can be edited again later.

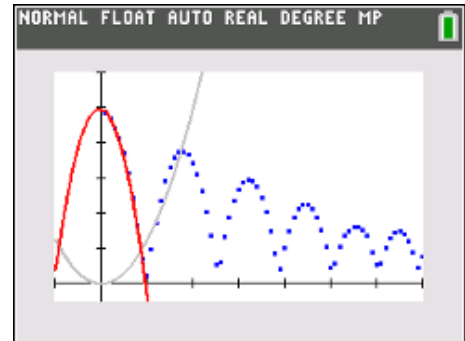


Question 3

How does this transformation affect the graph?

The last transformation, keeping all previous transformation is to edit the value of a in: $-aY_1(x - h) + k$

Once you have explored possible values of a , edit some of the previous transformations as necessary and try and provide the best match possible for the first drop of the bouncing ball.



Question 4

How does this transformation affect the graph?

Calculator Tip!



To get a closer view of the first ball bounce, use the Zoom option and use Zoom Box. Draw a box around the data and graph that you want to fill the screen. Edit the equation to get the best fit possible. When you are happy with your equation, use the Zoom – Memory option and select “Previous” to return to the original window settings.

Question 5

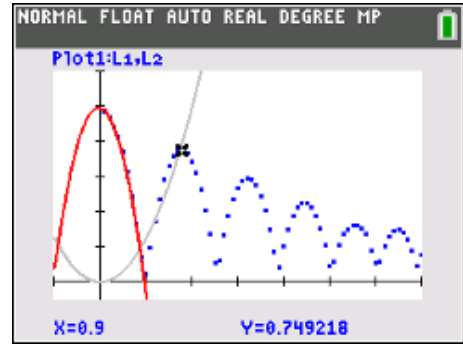
Write down the overall equation for your graph, using function notation, that models the first drop of the bouncing ball?

The remaining ball bounces can be modelled using the same approach as above.

Use Y_3 in terms of Y_1 to model the second ball bounce and so on with the remaining ball bounces.

The TRACE option can be used to determine specific Time and Position coordinates to assist in the transformations. Remember that the data is real so specific points may not represent the ideal location for the turning point on any given ball bounce / quadratic function.

The calculator screen shown opposite shows the approximate location of the peak height for the second ball bounce.



Calculator

Tip!



The TRACE option automatically stores the current X and Y coordinates in variables X and Y. These values can be recalled using $\boxed{2nd} \boxed{[rc]}$ and selecting the corresponding variable. This is a very efficient way to translate the function!

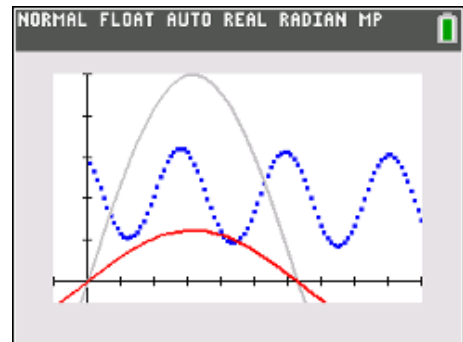
Question 6

Determine equations for the remaining ball bounces.

Pendulum

Run the program "Prob2". This will select the data for the pendulum swing problem, set up a scatter plot and enter the basic function in Y_1 .

In Y_2 enter the equation: $0.25Y_1$.



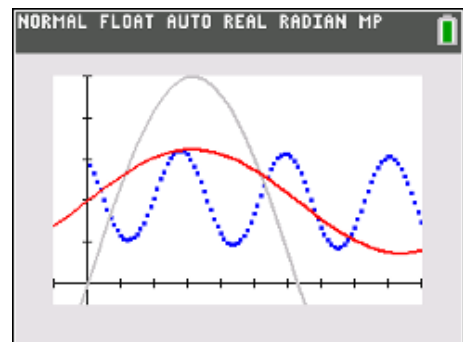
Question 7

How does this first transformation affect the graph?

The next transformation is of the form:

$$aY_1 + k$$

Explore the values of k that provide a reasonable transformation. Once again, this is approximate, all these transformations can be edited as the modelling function gets closer to the actual data.

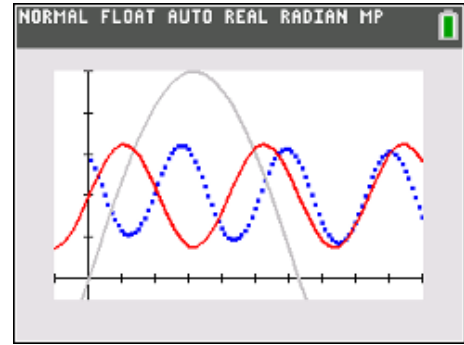


Once you are happy with the previous transformation explore and edit the following transformation:

$$aY_1(nx) + k$$

Start with a value of $n = 3$.

At this stage it is appropriate to edit the previous two transformations.

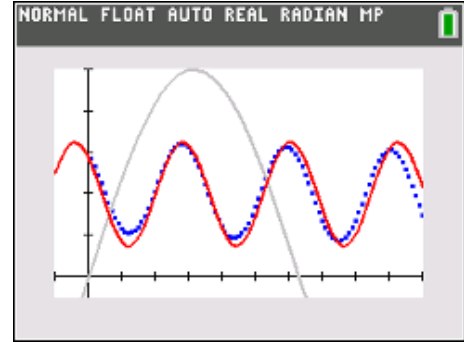


The final transformation for this function is:

$$aY_1(n(x-h)) + k$$

Make sure all parentheses are placed appropriately. There are numerous value of h that will work for modelling the pendulum. Identify one and work on the specific value.

As your function gets closer and closer to the data, edit each parameter accordingly.



Question 8

Record the function that best models the pendulum data.

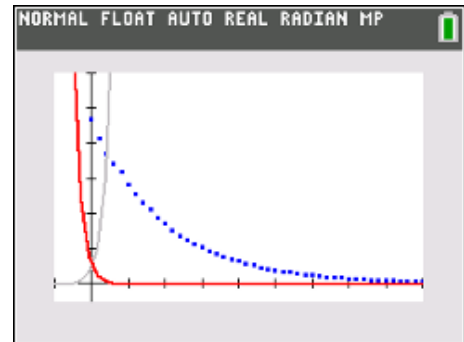
Capacitor

Run the Prob3 program. The electrical discharge of a capacitor has been recorded. A knowledge of electronics is not required for this task, however it is nice to know just how precisely electronics can align to mathematical equations!

An exponential function has been defined in: Y_1 .

Use Y_2 to explore transformations of the form:

$$f_2(x) = af_1(bx)$$



Question 9

Record the function that best models the discharge of a capacitor.