Transformations: Reflections

MATH NSPIRED

Math Objectives

- Students will identify a reflection as an isometry, also called a congruence transformation.
- Students will identify which properties are preserved in a reflection and which are not.
- Students will identify coordinates of an image that is reflected over the *x*-axis, the *y*-axis, and both axes.
- Students will generalize the relationship between the coordinates of a point and the coordinates of its reflection in the coordinate plane.
- Students will look for and make use of structure (CCSS Mathematical Practice).

Vocabulary

- pre-image
- reflection
- congruent figures
- image
- isometry
- transformation
- congruence transformation

About the Lesson

- Students will model reflections and identify the properties that are preserved in a reflection and those that are not. Then they will identify and generalize the coordinates of a triangle under reflections over the axes in the coordinate plane.
- As a result students will:
  - Reflect a triangle over a line and over the axes in the coordinate plane to develop their visualization and spatial sense of a reflection.
  - Describe the consequences of the reflection in terms of identifying those properties which are preserved and those which are not, and identify and generalize the coordinates of reflections in the coordinate plane.
  - Infer that a reflection does not alter any of the measurements of a reflected object, and as such, a reflection is an example of an isometry, or congruence transformation.
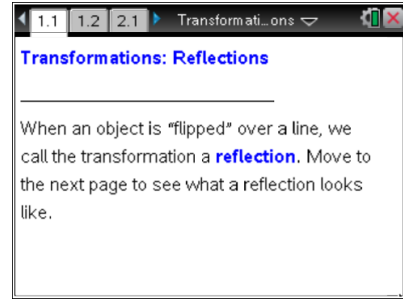
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- Use Class Capture to observe students' work as they proceed through the activity.
- Use Live Presenter to have a student illustrate how he/she used a certain tool.
- Use Quick Poll to assess students' understanding.

Activity Materials

- Compatible TI Technologies: ▦ TI-Nspire™ CX Handhelds, 📱 TI-Nspire™ Apps for iPad®, 💻 TI-Nspire™ Software

Transformations: Reflections

When an object is "flipped" over a line, we call the transformation a **reflection**. Move to the next page to see what a reflection looks like.

Tech Tips:

- This activity includes screen captures taken from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:
Student Activity
- Transformations_Reflections _Student.pdf
- Transformations_Reflections _Student.doc
TI-Nspire document
- Transformations_Reflections.tns

Discussion Points and Possible Answers

> **Tech Tip:** If students experience difficulty dragging a point, check to make sure that they have moved the cursor (arrow) until it becomes a hand (☝). Press `ctrl` ☝ to grab the point and close the hand (☝). When finished moving the point, press `esc` to release the point.
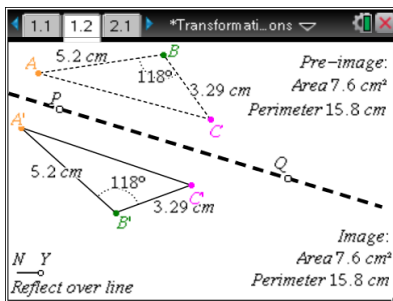
> **Tech Tip:** To move a point, tap on it once. This brings up a menu of options. Select the item you want to move and press Done. The point is now ready to move.

Move to page 1.2.

1. Predict what the reflected image of the triangle over the given line will look like. Draw your sketch in the space below.
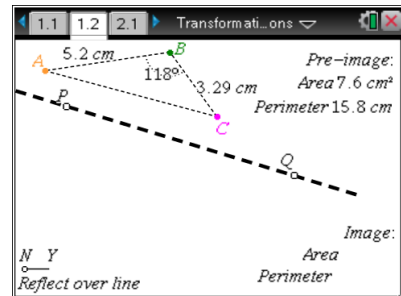
Answer:

Teacher Tip: Check to ensure that student sketches do not resemble a translation. Check to ensure that there is proper spacing of the vertices and slopes of the sides with respect to the line of reflection.

(At this point, the teacher should note spacing, slope, and placement, for discussion in the lesson, but not check student sketches to ensure accuracy. This is what the lesson is developing.)

2. Drag the reflection slider point so that it is below the letter *Y* (for Yes) to show the reflected image. How does the reflection compare to your prediction in problem 1? Redraw the reflection if necessary.

Answer: Discuss student responses.

> **Teacher Tip:** Make sure that students make corrections to their initial sketch if it does not match the reflection image.

3. Use page 1.2 to complete the table below.

	△ *ABC*	△ *A'B'C'*
Side length	*AB* = **6.6 cm**	*A'B'* = **6.6 cm**
Angle measure	∠*B* = **106°**	∠*B'* = **106°**
Side length	*BC* = **3.72 cm**	*B'C'* = **3.72 cm**
Area	**11.9 cm^2**	**11.9 cm^2**
Perimeter	**18.8 cm**	**18.8 cm**

4. Two figures are said to be congruent if they have the same size and same shape. Is △ *ABC* congruent to △ *A'B'C'*? Explain your reasoning.

Answer: Yes, they are congruent because they are the same size and same shape. There is no change in the side lengths or angle measurements.

5. Change the triangle by dragging one of its vertices and observe the changes in the reflected image. Would this change your answer to question 4? (Is △ *ABC* congruent to △ *A'B'C'*?)

Answer: Yes, they are still the same size and shape.

6. Drag point *P* or point *Q* to move the line of reflection and observe the changes in the reflected image. Would this change your answer to question 4? (Is △ *ABC* congruent to △ *A'B'C'*?)

Answer: Yes, they are still the same size and shape.

> **TI-Nspire Navigator Opportunity:** *Class Capture*
> **See Note 1 at the end of this lesson.**

7. An **isometry** is a transformation that produces an image that is congruent to the pre-image. Is a transformation using a reflection an isometry? Explain your reasoning.

Answer: Yes, because the pre-image and the image are congruent.

> **Teacher Tip:** This is an example of an isometry, because it is a congruence transformation. The measurements of all sides and angles are preserved by the reflection. Since all of the corresponding pairs of parts are congruent, the triangles are congruent. You may wish to have students use the measurement tools to investigate these relationships further. You may also choose to have your students draw segments joining the corresponding vertices to observe that each of these segments is perpendicular to and bisected by the line of reflection.

TI-Nspire Navigator Opportunity: *Live Presenter*
See Note 2 at the end of this lesson.

8. If the clockwise order of the vertices of the image and the pre-image is the same, the figures are said to have the same **orientation.**

   a. Do $\triangle ABC$ and $\triangle A'B'C'$ have the same orientation? Why or why not?

   Answer: No, they do not have the same orientation. The order is different because the triangle is "flipped" over. For example, if the clockwise order of the vertices of some triangle is *ABC*, the clockwise order of the reflected image will be *A'C'B'*.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 3 at the end of this lesson.

   b. Does the shape or location of the pre-image triangle affect your answer to question 8a? Does the location of the reflection line affect your answer to question 8a? Explain your answers.

   Answer: No, none of these things affects the orientation of the pre-image.

> **Teacher Tip:** This is one of the key concepts of reflections, namely orientation. The reflection is an isometry, or congruence transformation; however, the image will not have the same orientation as the pre-image. In other words, all properties, except for orientation, are preserved in a reflection.

9. Consider the properties of side length, angle measure, perimeter, area, and orientation.

    a. Which of these properties are preserved in a reflection?

    Answer: Length, perimeter, area, and angle measurement are preserved.

    b. Which of these properties are not preserved in a reflection?

    Answer: Orientation is not preserved.

> **Teacher Tip:** Would the answers change if the transformation were a translation? If your students are ready, ask the same question about rotation.
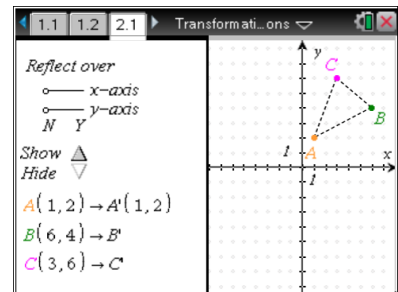

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 4 at the end of this lesson.

Move to page 2.1.

10. Predict the coordinates of △A'B'C' for each of the reflections in the table below. Coordinates of △ABC: A(1, 2); B(6, 4); C(3, 6)

    Sample Answer: See table below for answer when coordinates of △ABC are A(1, 2); B(6, 4); C(3, 6).

Reflection	Coordinates of A'	Coordinates of B'	Coordinates of C'
Over the x-axis	(1, –2)	(6, –4)	(3, –6)
Over the y-axis	(–1, 2)	(–6, 4)	(–3, 6)
Over both x-axis and y-axis	(–1, –2)	(–6, –4)	(–3, –6)

After you make your predictions, select △ to show the coordinates to check your results.

11. Select the Hide ▽ arrow to hide the coordinates of the image.

   a. Change the vertices of △ABC so that each vertex is in a different quadrant and record the new coordinates of the vertices.

   Sample Answer: △ABC: A(–1, 4), B(5, –2), C(3, 6)

   b. For these new vertices, again predict the coordinates of △A'B'C' for each of the reflections in the table below. After you make your predictions, select △ to show the coordinates to check your results.

   Sample Answer: See table below for previous coordinates.

Reflection	Coordinates of A'	Coordinates of B'	Coordinates of C'
Over the x-axis	(–1, –4)	(5, 2)	(3, –6)
Over the y-axis	(1, 4)	(–5, –2)	(–3, 6)
Over both <u>x</u>-axis and y-axis	(1, –4)	(–5, 2)	(–3, –6)

> **Teacher Tip:** Encourage students to carefully observe the signs. It also would help if the absolute values of all the coordinates of the pre-image are different as in the sample answer included here. Then, there will be a clearer distinction between what is changing and what is not changing in the various reflections. Encourage students to investigate and generalize reflections separately.

12. Generalize your results: If the coordinates of point P are (x, y), identify the coordinates of point P' if it is reflected over:

   a. the x-axis:

   Answer: $P'(x, -y)$

   b. the y-axis:

   Answer: $P'(-x, y)$

   c. both the x-axis and the y-axis:

   Answer: $P'(-x, -y)$

Teacher Tip: You also may want to have students write out the generalization in words. If a point is reflected over the *x*-axis, the *x*-coordinate of the image remains the same as the pre-image and the *y*-coordinate is the opposite of the pre-image. If a point is reflected over the *y*-axis, the *x*-coordinate of the image is the opposite of the pre-image and the *y*-coordinate remains the same as the pre-image. If a point is reflected over both axes, both coordinates of the image are opposite of the pre-image.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 5 at the end of this lesson.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students can:

* Define isometry or congruence transformation and explain why a reflection is an example of one.
* Recognize that length, perimeter, area, and angle measurement are preserved in a reflection, while orientation is not preserved.
* Identify coordinates of an image that is reflected over the *x*-axis, the *y*-axis, and both axes.
* Generalize the relationship between the coordinates of a point and the coordinates of its reflection in the coordinate plane.

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Note 1

Questions 5–6, Class Capture: Take Class Captures while students are answering questions 5 and 6 so that they can observe more examples before making a conclusion about whether the image and pre-image are congruent.

Note 2

Question 6, Live Presenter: Make someone the Live Presenter and as noted in the Teacher Tip above, have the student draw segments joining the corresponding vertices to observe that each of these segments is perpendicular to and bisected by the line of reflection.

Note 3

Question 8a, Quick Poll: Collect students' responses to question 8a with a Yes/No Quick Poll. Discuss their responses.

Note 4

Question 9, Quick Poll: Send students the following Always/Sometimes/Never

Quick Polls:

A translation _____ preserves orientation.

Answer: A translation <u>always</u> preserves orientation.

A rotation _____ preserves orientation.

Answer: A rotation <u>sometimes</u> preserves orientation.

Note 5

Question 12, *Quick Poll*: Send students the following Open Response Quick Polls:

Given the coordinates of *A*(-3,0), what are the coordinates of *A*' when △*ABC* is reflected about:

the *x*-axis.	**Answer:** (-3,0)
the *y*-axis.	**Answer:** (3,0)
both the *x*-axis and the *y*-axis.	**Answer:** (3,0)