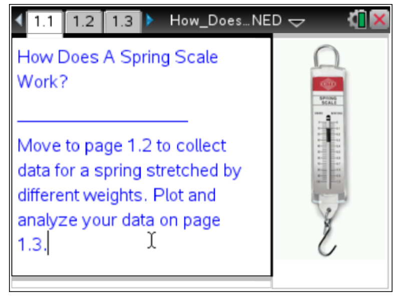
How Does a Spring Scale Work?



Math Objectives

- Students will discover that the constant of proportionality between two proportional quantities is the unit rate.
- Students will discover that the unit rate is the slope of the line.
- Students will compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in similar or different units (CCSS).
- Make sense of problems and persevere in solving them (CCSS Mathematical Practice).
- Look for and express regularity in repeated reasoning (CCSS Mathematical Practice).

Vocabulary

- ratio
- proportion
- constant of proportionality
- slope of a line
- unit rate

About the Lesson

- This lesson involves students using the simulation of how a spring scale works to record and graph the direct proportional relationship between the loaded weight and the spring's stretch.
- As a result, students will:
  - Observe and analyze changes in the stretch of the spring as different weights are loaded onto the spring.
  - Compute the constant of proportionality for the weight-to-stretch relationship.
  - Compute the unit rate associated with the ratio of weight to spring's stretch. The unit rate is equal to the strength of the spring scale.
  - Determine that for directly proportional quantities, the constant of proportionality is equal to the unit rate of change associated with the quantities and is equal to the slope of the line of best fit for the data.

Note: The activity simulates a well known physics phenomena called Hooke's Law that states the extension of an elastic spring is directly proportional to the load applied to the spring.

TI-Nspire™ Navigator™

- Send and collect a file.
- Use Class Capture to monitor student work.
- Use Live Presenter to let students demonstrate their work.
- Use Quick Poll to assess students' understanding.
- Use Class Analysis and Slide Presentation to review students' answers to questions.

Activity Materials

- Compatible TI Technologies: TI- Nspire™ CX Handhelds, TI-Nspire™ App for iPad®, TI-Nspire™ Software

Tech Tips:

- This activity includes screen captures taken from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:

Student Activity

- How_Does_a_Spring_Scale_Work_Student.pdf
- How_Does_a_Spring_Scale_Work_Student.doc

TI-Nspire document

- How_Does_a_Spring_Scale_Work.tns
- How_Does_a_Spring_Scale_Work_Assessment.tns (optional)

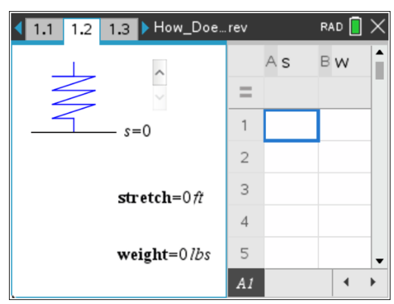
Discussion Points and Possible Answers

Tech Tip: If students experience difficulty dragging a point, check to make sure that they have moved the cursor until it becomes a hand (👆) getting ready to grab the point. Then press ctrl 👆 to grab the point and close the hand (✊).

Teacher Tip: To avoid student confusion about independent and dependent variables, teacher should clarify that in this activity we do not investigate how far any given weight can stretch the spring. This activity explores how the spring scale works. Spring scales allow us to use the length of the spring stretch to determine the weight. In this context, the weight is the dependent variable, and the stretch is the independent variable. Also, this choice is made due to the fact that the ratio weight-to-stretch is the spring constant (strength of the spring) and has been given, by convention, this physical meaning.

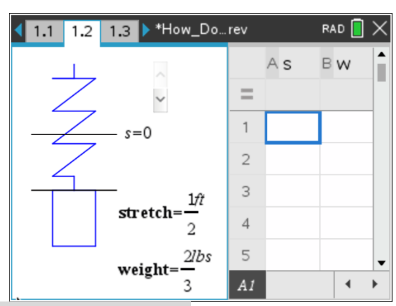
Tech Tip: Be sure students know how to add weight to the spring by using the up arrow of the minimized slider. Have students notice the relationship between the weight and the stretch of the spring. If needed, demonstrate the use of the slider.

Move to page 1.2.



1. We will stretch the simulated spring from its original length by loading different weights. Select the up arrow to add weight to the spring.

   a. Record the measured weight for each given spring's stretch in the second column of the table below.



Tech Tip: To add or modify data in a spreadsheet cell, double-tap the cell.

Answer:

Stretch (s), ft	Weight (w), lbs	Ratio, $\frac{w}{s}$, lbs/ft
$\frac{1}{8}$ ft	$\frac{1}{6}$ lbs	$\frac{\frac{1}{6}}{\frac{1}{8}} = \frac{1}{6} \cdot \frac{8}{1} = \frac{8}{6} = \frac{4}{3}$ lbs/ft
$\frac{1}{4}$ ft	$\frac{1}{3}$ lbs	$\frac{\frac{1}{3}}{\frac{1}{4}} = \frac{1}{3} \cdot \frac{4}{1} = \frac{4}{3}$ lbs/ft
$\frac{3}{8}$ ft	$\frac{1}{2}$ lbs	$\frac{\frac{1}{2}}{\frac{3}{8}} = \frac{1}{2} \cdot \frac{8}{3} = \frac{8}{6} = \frac{4}{3}$ lbs/ft
$\frac{1}{2}$ ft	$\frac{2}{3}$ lbs	$\frac{\frac{2}{3}}{\frac{1}{2}} = \frac{2}{3} \cdot \frac{2}{1} = \frac{4}{3}$ lbs/ft

2. What do you observe as you add weight to the spring?

Sample Answers: As weight is added to the spring, the spring stretches: the heavier the weight, the longer the stretch.

TI-Nspire Navigator Opportunity: *Class Capture and Live Presenter*
See Note 1 at the end of this lesson.

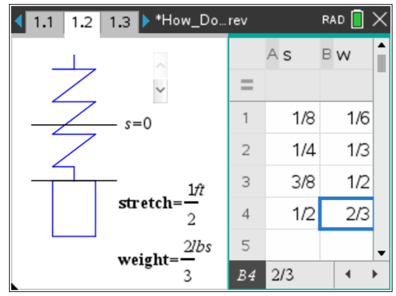
3. If the stretch doubles, what happens to the weight? Give two examples to support your conclusion.

Answer: When the stretch doubles, the weight doubles. For example, stretch changed from $\frac{1}{8}$ ft to $\frac{1}{4}$ ft, the weight changed from $\frac{1}{6}$ lbs to $\frac{1}{3}$ lbs. Another example, stretch changed from $\frac{1}{4}$ ft to $\frac{1}{2}$ ft, the weight changed from $\frac{1}{3}$ lbs to $\frac{2}{3}$ lbs.

Teacher Tip: Ask students to explain operations with fractions, for example, $\frac{1}{6} \times 2 = \frac{2}{6} = \frac{1}{3}$.

4. Enter your data into the Lists & Spreadsheet on Page 1.2.

   Answer: See screen shot.



> **Tech Tip:** Students can move the cursor to the right side of the page and select any cell to start data entry, or use ctrl tab to switch to Lists & Spreadsheet.

5. Return to the table on the previous page of this worksheet. Simplify the ratio of the weight (*w*) to the stretch (*s*) for each entry in the table in order to determine unit rate. Record the ratios in the third column and show your calculations.

   Answer: See table on previous page.

> **Teacher Tip:** This is a good place for students to practice division of fractions by hand. Discuss with students what is meant by "ratio" and how is that different from "unit rate".

> **Tech Tip:** Students can use the Lists & Spreadsheet to verify their calculations. You might want to demonstrate how to do calculations in the Lists & Spreadsheet. Select the cell c1 and type =b2/a2. Repeat calculations in cells c2 – c4.

6. What are your observations about these unit rates?

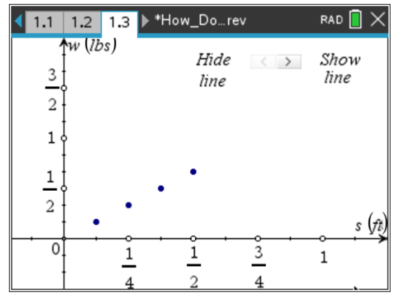
   Answer: The unit rates are equal.

7. Is there a relationship between the stretch of the spring and the loaded weight? If so, explain what it is, and why you think it exists.

   Answer: Since the unit rates are equal, the quantities are proportional.

> **Teacher Tip:** This would be a good time to remind students that unit rate is also called *constant of proportionality*.

Move to page 1.3.

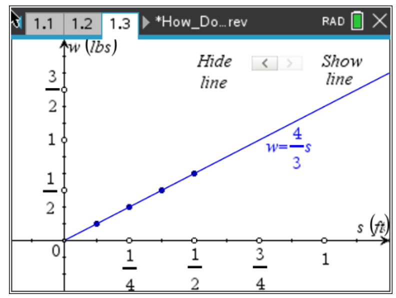


8. Describe what the scatter plot on this page represents.

   <u>Sample Answers:</u> The scatter plot represents a relationship between the stretch of the spring and the weight on the spring. Each point shows given stretch and corresponding weight.

9. What are some of the interesting features of the scatter plot?

   <u>Answer:</u> The points lie in a line. The points go up as you move from left to right.



10. Select the "show line" arrow to display the line of best fit and an equation that models the relationship between the weight on the spring and the stretch of the spring. What observations can you make?

    <u>Answer:</u> The line starts at zero and goes through all points. The number in front of s in the equation is equal to the ratio we found earlier.

11. a. What is the unit rate for any given weight to the spring's stretch $\left(\frac{w}{s}\right)$? What evidence do you have for this?

    <u>Answer:</u> The unit rate is $\frac{4\text{ lbs}}{3\text{ ft}}=\frac{4}{3}$ lbs/ft, and it is the same for all entries in the table as calculated earlier.

    b. What is the slope of the line that goes through the points? On what are you basing your response?

    <u>Answer:</u> The slope of the line is $\frac{4\text{ lbs}}{3\text{ ft}}=\frac{4}{3}$ lbs/ft. The slope is rise over run which is a unit rate that we calculated. It is also a number in front of *s* in the equation $w=\frac{4}{3}s$

    c. What do you notice about the unit rate and the slope? What does it tell you?

    <u>Answer:</u> The slope of the line has the same numerical value and the same units as the unit rate of weight to stretch. It tells us that the relationship is linear and proportional, $\frac{w}{s}=\frac{4}{3}$, so $w=\frac{4}{3}s$.

Teacher Tip: You might want to have students share how they determined the slope. Depending on the students' background, some might have noted that the coefficient of s in the formula represents the slope. Others might have calculated the (change in w)/(change in s) between two points. Still others might have counted rise over run on the grid, though this would make for an interesting discussion how to accomplish this when one scale is in eighths, while the other is in sixths.

This is also a good place to introduce rate of change and compare that to unit rate. Let students notice that for every four lbs of weight added to the spring, the stretch will increase by 3 feet. However, this is only theoretical since the spring will probably break or lose its elasticity before then. This first interpretation is a rate of change interpretation of the slope. Another interpretation is the unit rate interpretation of the slope: for every 4/3 lbs of weight added, the spring will stretch by 1 foot. Both interpretations should be made explicit to the students.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 2 at the end of this lesson.

12. Explain why the line *begins* at the origin rather than going *through* the origin.

Answer: This particular experiment begins when there is no weight on the spring, so it is not stretched. Thus initial stretch is zero and initial weight is zero. When the weight is loaded onto the spring, the spring can only get longer, so the stretch is always positive. The weight cannot be negative.

Teacher Tip: Discuss with the students how a physical experiment could be modified to include "negative stretch" or compression and take into account the direction of the force on the spring: pulling down to cause stretch or pushing up to cause compression. In this modified experiment, the line will go through the origin.

13. a. Using your findings, determine the strength of the spring scale (the spring constant). Indicate units of measurements.

Answer: $\frac{4}{3}$ lbs/ft

Teacher Tip: The spring constant is measured in lbs/ft and is equal to the weight that the spring can hold when stretched by 1 ft. Students can approach this question in different ways, for example, they can use the equation of the line to calculate the weight needed to stretch the spring by 1 ft: $w = \frac{4 \text{ lbs}}{3 \text{ ft}}(1 \text{ ft}) = \frac{4}{3} \text{ lbs}$, thus spring constant is $\frac{4}{3}$ lbs per foot.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 3 at the end of this lesson.

b. Write a summary statement comparing the strength of the spring scale to the unit rate of change and comparing the strength of the spring scale to the slope of the line.

Answer: The weight changes at a rate of $\frac{4}{3}$ lbs per foot, e.g. for each $\frac{4}{3}$ lbs loaded on the spring, the spring stretches 1 foot. Thus the spring constant, by definition, is equal to the unit rate. When plotting this relationship, the unit rate is represented by the slope of the line on the graph of weight vs stretch. As a result, the spring constant, the unit rate, and the slope of the line have the same numerical values and the same units. The spring constant is the constant of proportionality of this relationship.

Teacher Tip: It is recommended that students share their explanations. Encourage different students to present various explanations of why these three quantities are the same. Make sure students include units of measurements in their explanations.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students are able to understand:

- The constant ratio of two proportional quantities is known as the constant of proportionality between two proportional quantities and when written in simplest terms is equal to the unit rate
- The unit rate is equal to the slope of the line for direct proportion.
- How to practice skills of dividing fractions by using the fraction key and parenthesis to represent a fraction as a numerator or a denominator in a ratio.
- Why the units are important.
- How to use real springs and weights and let students determine spring constant experimentally.

Assessment

1. Given a spring with spring constant equal to $\frac{1}{2}$ lbs/ft, find the weight needed to stretch the spring by $\frac{3}{4}$ ft.

   Answer: $w = \frac{1}{2}\ \text{lbs/ft} \times \frac{3}{4}\ \text{ft} = \frac{1}{2} \times \frac{3}{4} \frac{\text{lbs} \cdot \text{ft}}{\text{ft}} = \frac{3}{8}\ \text{lbs}$.

2. Given a spring with spring constant equal to $\frac{1}{2}$ lbs/ft, find the amount of stretch that will be caused by $\frac{1}{4}$ lb.

   Answer: $\frac{1}{4}\ \text{lbs} = \frac{1}{2}\ \text{lbs/ft} \times s\ \text{ft}$, so $s = \dfrac{\frac{1}{4}\ \text{lbs}}{\frac{1}{2}\ \frac{\text{lbs}}{\text{ft}}} = \frac{1}{4} \times \frac{2}{1}\ \text{lbs}\frac{\text{ft}}{\text{lbs}} = \frac{1}{2}\ \text{ft}$.

3. Will the numerical value of the spring constant change if you change units to grams and centimeters?

   Answer: Yes. The numerical value of spring constant changes, the new value can be found using unit conversion: $\frac{1}{2}\frac{\text{lbs}}{\text{ft}}\frac{1\ \text{ft}}{30.5\ \text{cm}}\frac{454\ \text{g}}{1\ \text{lb}} = 7.44\ \text{g/cm}$

4. What is the numerical value of the spring constant $\frac{1}{2}$ lbs/ft in grams/cm? Given: 1 lb = 454 g, 1 ft = 30.5 cm.

   Answer: The numerical value of spring constant can be found using unit conversion:
   $\frac{1}{2}\frac{\text{lbs}}{\text{ft}}\frac{1\ \text{ft}}{30.5\ \text{cm}}\frac{454\ \text{g}}{1\ \text{lb}} = 7.44\ \text{g/cm}$

5. Does the strength of the spring change when you change the units?

   Answer: No. The numerical value of spring constant changes, but the strength of the spring remains the same.

TI-Nspire Navigator Opportunity: *Class Analysis and Slide Show*
See Note 4 at the end of this lesson.

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Note 1

Question 2, Name of Feature: Class Capture and Live Presenter

Ask students to choose different weights to display on the handhelds, and use *Class Capture* to display the springs that are stretched by different amounts. Use *Live Presenter* to give control to students to demonstrate the use of the slider while explaining their observations about the weight and the stretch.

Note 2

Question 11, Name of Feature: Quick Poll

A *Quick Poll* can be given after students worked on three-part question 11. For questions 11a and 11b use *Open Response* option. You can use True – False option for question 11c, for example, "The slope of the line is equal to the ratio of weight to stretch, true or false?"

Note 3

Question 13, Name of Feature: Quick Poll

A *Quick Poll* can be given to students for questions 13a to collect results and also to check vocabulary and use of units of measurements.

Note 4

Assessment, Name of Feature: Class Analysis and Slide Show

Send students the assessment document (How_Does_a_Spring_Scale_Work_Assessment.tns) to answer questions 1) – 5) provided in the Assessment section. Collect the document and conduct a Slide Presentation to go over the answers. Ask students to explain their answers and calculations.

