Polar Necessities
ID: 12559

Activity Overview
Students will explore what is necessary to understand about polar equations for calculus. Students will graphically and algebraically find the slope of the tangent line at a point on a polar graph. Multiple-choice questions are self-check so students can check their understanding. Finding the area of a region of a polar curve will be determined using the area formula.

Topic: Polar Equations
- *Find the slope of a polar equation at a particular point.*
- *Find the area of polar equation.*

Teacher Preparation and Notes
- *To graph a polar equation on a Graphs & Geometry application page, select **MENU > Graph Type > Polar**. Alternatively, use 'right-click' ([ctrl] + [menu]) when on the entry line of a Graph & Geometry page.*

- *On self-check questions, students can get immediate feedback by pressing [menu] **Check Answer** (or [ctrl] + ▲). With TI-Nspire Navigator Teacher Edition software, teachers can change self-check questions to exam mode so students cannot check their answer. On any question click the Teacher Tool Palette and select Question Properties. Change the Document Type from Self-Check to Exam.*

- *Polar equations are a BC topic. AB teachers may enjoy using this activity after the AP* exam or using with students in your AB class who want to prepare for the BC exam. After completing the activity, students should be more successful with AP questions like multiple-choice 98BC19, 73BC40, and free response 05BC2, 03BC3, 93BC4, 90BC4, 84BC5.*

- *Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.*

- ***To download the student and solution TI-Nspire documents (.tns file) and student worksheet, go to education.ti.com/exchange and enter "12559" in the keyword search box.***

Associated Materials
- *Polar_Student.doc*
- *Polar.tns*
- *Polar_Soln.tns*

Suggested Related Activities
To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *Polar Graphs (TI-Nspire technology) — 16123*
- *Cardioid Patterns – Discover Using Graphs (TI-Nspire technology) — 10149*

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Part 1 – Plotting Coordinates & Exploring Polar Graphs

Students begin the activity by plotting points on a polar graph. On page 1.3, students can grab and drag the end of the arrow to 'point' to the location they are plotting. This should be a refresher of polar coordinates for most students.
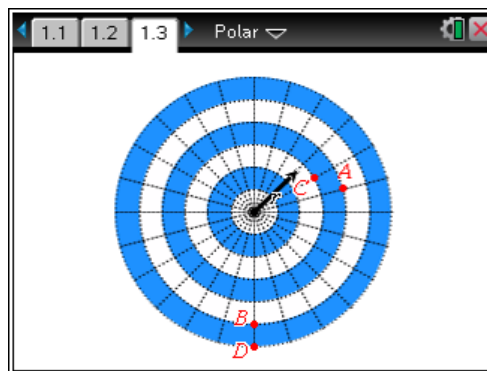

> **TI-Nspire Navigator Opportunity:** *Screen Capture*
> **See Note 1 at the end of this lesson.**

Discussion Questions

- *What do you think it means to have a negative angle, like $\left(-\dfrac{\pi}{3}, 3\right)$?*

- *What about if r was negative? For example, move to $\left(\dfrac{\pi}{2}, -6\right)$.*

On page 1.6, students will plot the *r*-value for each *θ* for the given function. This page can be used with other polar equations, as well. By toggling over to the graph (ctrl + tab), students can check their points by graphing the polar equation on this screen.
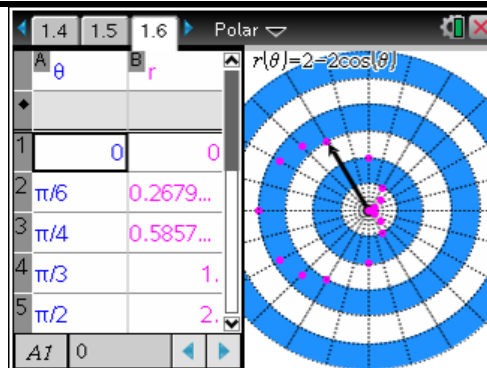
Page 2.2 has students explore polar graphs. The equation **r1** can be changed by double-clicking. Some polar graphs complete their cycle in π radians instead of 2π. This will be helpful to have students notice when they move on to finding the area.
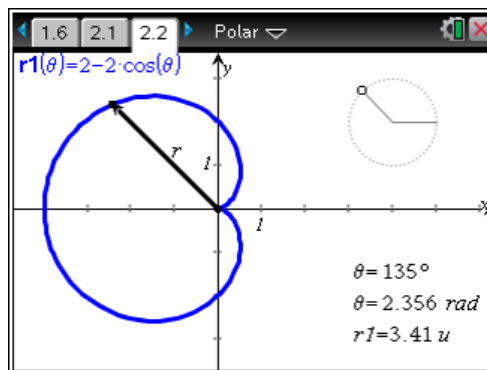
Student Solutions

1. See the screenshot of page 1.3 at above right.

2. If $r(\theta) = \cos(\theta)$, $r\left(\dfrac{\pi}{3}\right) = 0.5$.

3. a cardioid

4. circle is in the form $r = a$, where *a* is a constant
    polar rose with even petals is in the form $r(\theta) = a \cdot \sin(n\theta)$, where *n* is even
    polar rose with odd petals is in the form $r(\theta) = a \cdot \sin(n\theta)$, where *n* is odd
    a limaçon with an inner loop comes form $r(\theta) = b + a \cdot \cos(\theta)$, where $b < a$

> **TI-Nspire Navigator Opportunity:** *Screen Capture* **and/or** *Live Presenter*
> **See Note 2 at the end of this lesson.**

Part 2 – Slopes of Polar Graphs

Students recall what they know about the slope of a line and find the locations of the vertical and horizontal tangents. Problem 5 introduces the topic of slope. It will be of value to have students consider the meaning of $\dfrac{dr}{d\theta}$.
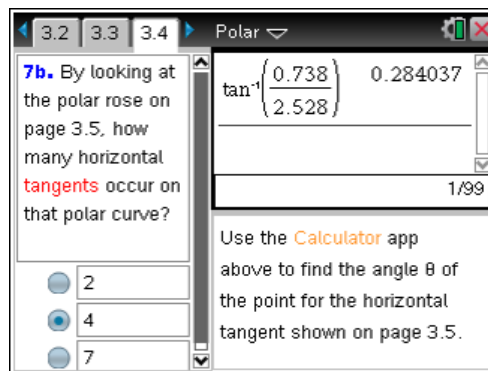
TI-Nspire Navigator Opportunity: *Quick Poll*
**See Note 3 at the end of this lesson.**

Discussion Questions

- *What does is mean if $\dfrac{dr}{d\theta}$ is negative?*

- *On page 3.5, where on the polar rose is $\dfrac{dr}{d\theta}$ negative?*

Problems 6 and 7 lead students to see that the slope of the tangent is $\dfrac{dy}{dx} = \dfrac{\dfrac{d(r \cdot \sin\theta)}{d\theta}}{\dfrac{d(r \cdot \cos\theta)}{d\theta}}$.

After Problem 7c has been answered, students can grab and drag the point of tangency to observe the slope for other values of *x* and *y*.

Student Solutions

5. $\dfrac{dy}{dx}$
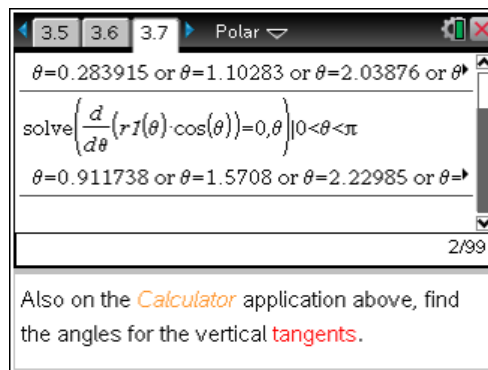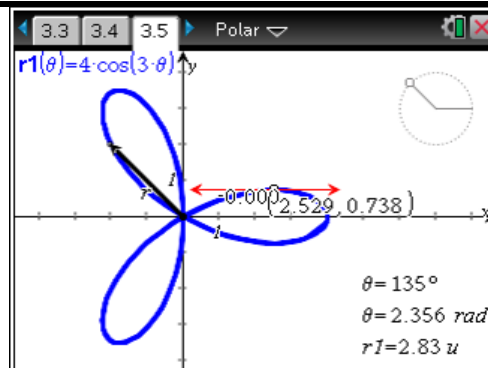
6. $x = r \cdot \cos\theta$ and $y = r \cdot \sin\theta$

7. a. $\dfrac{dy}{d\theta} = 0$, $\dfrac{dx}{d\theta} \neq 0$

   b. There are four horizontal tangents.

   c. $\tan^{-1}\left(\dfrac{0.738}{2.528}\right) \approx 0.284$

   d. solve(d/dθ(r1(θ)*cos(θ))=0,θ)|0<θ<π
      θ = 0.912 or θ = 1.571 or θ = 2.2299 or θ = 3.14159

8. $\left.\dfrac{d(r1(\theta) \cdot \sin(\theta))}{d\theta}\right|_{\theta=\frac{2\pi}{3}} = -2$, $\left.\dfrac{d(r1(\theta) \cdot \cos(\theta))}{d\theta}\right|_{\theta=\frac{2\pi}{3}} = -2\sqrt{3}$, $\dfrac{dy}{dx} = \dfrac{-2}{-2\sqrt{3}} = \dfrac{\sqrt{3}}{3} \approx 0.557$

Part 3 – Area of Polar Graphs

In this part of the activity, the formula for the area of a polar curve is given and students will use this to find the area of the shaded region using CAS. One of the most difficult parts of this topic is finding the limits of integration.

Other equations can be explored using the rotating slider by changing the equation on the screen. The shaded region will remain from 0 to $\frac{\pi}{3}$. This can be changed on page 4.1. On the right side of the screen is a minimized spreadsheet. Expand this application. Enter a new value in Cell A1 and the region from 0 to that value will be shaded.
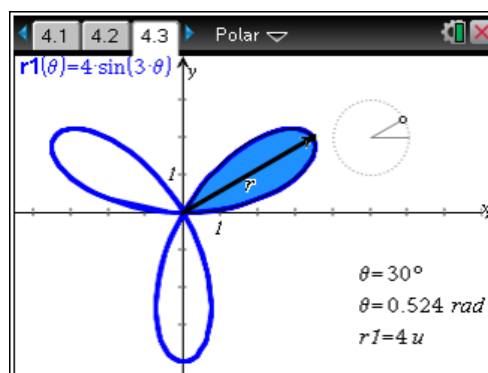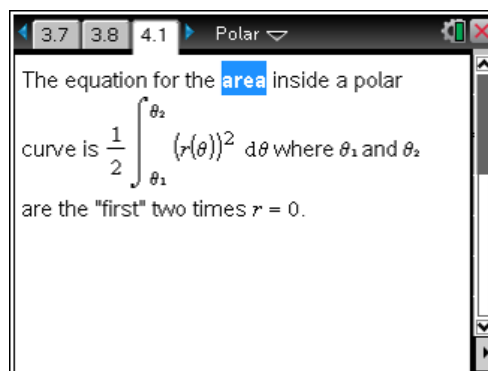

Students should know the polar area formula for exams. If students are asked to evaluate the integral, it is likely that it will be calculator active. Non-calculator questions will have them set up these types of problems.

If time allows, explain how this formula is derived from triangles instead of rectangles, like Riemann sums.

Student Solutions

9. 0 to $\frac{\pi}{3}$

10. 4.189

TI-Nspire Navigator Opportunity: *Collect from Class*
See Note 4 at the end of this lesson.

TI-Nspire Navigator Opportunity

Note 1
Part 1, *Screen Capture*
Do a whole class screen capture. Press ctrl+0, or change Zoom Percentage under View to "Fit All." Use question 1 or make up your own coordinates for the students to move the vector to. Refresh the screen to make sure everyone got to the correct location. You could make this a race.

Note 2
Part 1, *Screen Capture* and/or *Live Presenter*
Screen Capture can be used here and throughout the lesson to ensure students are able to follow directions on graphing polar equations. It may be helpful for students to show the different equations that students graph on page 2.2 in order to answer Problem 4. You may choose to pick a student (or group of students) as a live presenter to demonstrate the activity to the class.

Note 3
Part 2, *Quick Poll*
Use Quick Poll for documented formative assessment with pages 2.1, 3.1–3.4. This can help to ensure students are staying on task.

Note 4
Part 3, *Collect from Class*

From the Class workspace, collect the file from class using this icon or clicking the arrow to the right of the name of the file. Also use this arrow to save it to the Portfolio, and Open in the Review workspace. Discuss the students' answers to Questions 8, 9, and 10 that they submitted on their TI-Nspire handheld.