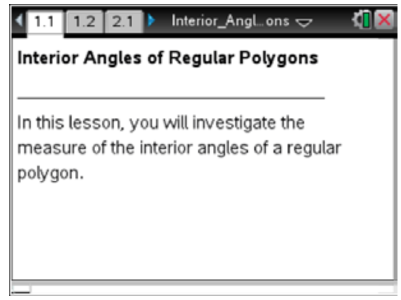



## Math Objectives

- Students will find the central angle measure of a regular polygon.
- Students will relate the sum of the interior angles of a triangle to the sum of the interior angles of a regular polygon.
- Students will apply geometric representations of the expressions ($n$ – 2)180 and 180$n$ – 360 to determine the measure of the interior angles of a regular polygon.

## Vocabulary

- central angle
- base angle
- interior angle
- isosceles triangle
- regular polygon

## About the Lesson

- This lesson involves changing the number of sides of a regular polygon.
- As a result students will:
  - Observe the consequences of this manipulation on the central angle.
  - Infer the relationship between the central angle and the number of sides of a regular polygon.
  - Infer the relationship between the base angles of the isosceles triangles and the measure of an interior angle.
  - Deduce the geometric and algebraic equivalence of the expressions ($n$ – 2)180 and 180$n$ – 360, which can be used to find the interior angle sum of all regular and irregular convex polygons.

## TI-Nspire™ Navigator™ System

- Send out the *Interior_Angles_of_Regular_Polygons.tns* file.
- Monitor student progress using Class Capture.
- Use Live Presenter to spotlight student answers.

## Activity Materials

- Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software

- **Tech Tips:** This activity includes screen captures taken from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

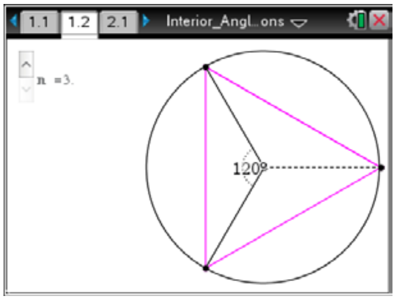
**Lesson Materials:**
*Student Activity*
- Interior_Angles_of_Regular_Polygons_Student.pdf
- Interior_Angles_of_Regular_Polygons_Student.doc

*TI-Nspire document*
- Interior_Angles_of_Regular_Polygons

## Discussion Points and Possible Answers

**Move to page 1.2.**

1. Use the polygons and central angle measurements to complete the following table.




**Tech Tip:** Tap the up arrow to change the number of sides of the polygon. **Longer taps may be necessary.**

| **Regular Polygon** | **# of Sides** | $\frac{\mathbf{360°}}{\mathbf{\#\ of\ sides}} =$ | **Central Angle Measure** |
|---|---|---|---|
| Triangle | 3 | $\frac{\mathbf{360°}}{\mathbf{3}} =$ | 120° |
| Quadrilateral | 4 | $\frac{\mathbf{360°}}{\mathbf{4}} =$ | 90° |
| Pentagon | 5 | $\frac{\mathbf{360°}}{\mathbf{5}} =$ | 72° |
| Hexagon | 6 | $\frac{\mathbf{360°}}{\mathbf{6}} =$ | 60° |
| *n*-gon | *n* | $\frac{\mathbf{360°}}{n} =$ | $\frac{360°}{n}$ |

a. Each regular polygon is divided into triangles. What type of triangles are they? Why?

**<u>Sample Answers:</u>** Each polygon, except the hexagon, is divided into isosceles triangles. All radii of a circle are congruent. Since 2 of the legs of each triangle are radii, the triangles must be isosceles. The hexagon is divided into equilateral triangles, which have all three sides the same length.

b. For each regular polygon, what is the relationship between these triangles? Why?

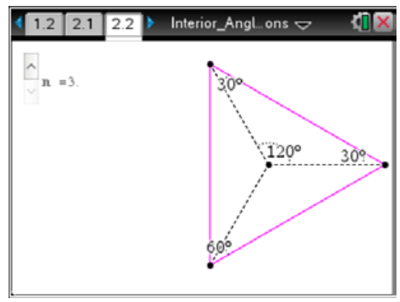
**<u>Sample Answers:</u>** For each regular polygon, all inscribed isosceles triangles are congruent. All radii of a circle are congruent, so 2 sides of each triangle are congruent. The sides of each regular polygon are also congruent. Therefore, the triangles are congruent according to the SSS congruence theorem.

c. Use the pattern in the table to find the central angle measure of a regular octagon (8 sides).

**Sample Answer:** $\frac{360°}{8} = 45°$

**TI-Nspire Navigator Opportunity: *Quick Poll***
**See Note 1 at the end of this lesson.**

**Move to page 2.2.**




2. Madeline created the following table to explore the relationship between the number of triangles and the angle measurements in regular polygons.

a. Complete Madeline's table.

| Regular Polygon | # of Sides | # of Triangles | # of Triangles(180°) – 360° = | Sum of Interior Angles | Sum of Base Angles of 1 Δ |
|---|---|---|---|---|---|
| Triangle | 3 | 3 | 3(180°) – 360° = | 180° | 60° |
| Quadrilateral | 4 | 4 | 4(180°) – 360° = | 360° | 90° |
| Pentagon | 5 | 5 | 5(180°) – 360° = | 540° | 108° |
| Hexagon | 6 | 6 | 6(180°) – 360° = | 720° | 120° |
| *n*-gon | *n* | *n* | $n(180°) - 360°$ = | $n(180°) - 360°$ | $180° - \frac{360°}{n}$ |

b. When the number of sides in a regular polygon increases by 1, why does the interior angle sum increase by 180°?

**Sample Answer:** When the number of sides in a regular polygon increases by 1, the number of triangles that can be drawn within the polygon also increases by 1. Since the sum of the interior angles of a triangle is 180°, the interior angle sum increases by 180°.

**Teacher Tip:** Make sure students understand the construction is causing this to happen. The polygon is divided into triangles in such a way that they meet in the center and each extends to have a base that is one of the sides of the polygon. Ask students why they should subtract 360°.

c. Use the pattern in the table to find the sum of the base angles of an isosceles triangle drawn from the center of a regular nonagon (9 sides).

**Sample Answer:** $180° - \frac{360°}{9} = 140°$

**Teacher Tip:** This may require some discussion while completing the table.

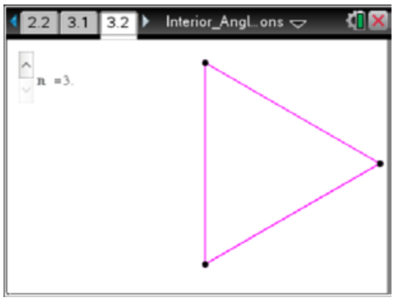
d. When congruent isosceles triangles are drawn from the center of a regular polygon, why is the base angle sum of any one of the isosceles triangles equivalent to the measure of an interior angle of the polygon?

**Sample Answer:** Since the isosceles triangles drawn from the center of a regular polygon are congruent, their corresponding parts are also congruent. Therefore, the base angles of all triangles are congruent. Since the adjacent base angles of adjacent triangles form an interior angle, the sum of the base angles is equivalent to the measure of an interior angle.

**TI-Nspire Navigator Opportunity: *Quick Poll***
**See Note 2 at the end of this lesson.**

**Move to page 3.2.**

3. Joshua created a different table to explore the relationship between inscribed triangles and the interior angle measurements in regular polygons.



a. Complete Joshua's table.

| Regular Polygon | # of Sides | # of Triangles | # of Triangles (180°) = | Sum of Interior Angles | $\frac{\text{Interior Angle Sum}}{\text{\# of Sides}}$ = | Interior Angle Measure |
|---|---|---|---|---|---|---|
| Triangle | 3 | 1 | 1(180°) = | 180° | $\frac{180°}{3}$ = | **60°** |
| Quadrilateral | 4 | 2 | 2(180°) = | 360° | $\frac{360°}{4}$ = | **90°** |
| Pentagon | 5 | 3 | 3(180°) = | 540° | $\frac{540°}{5}$ = | **108°** |
| Hexagon | 6 | 4 | 4(180°) = | 720° | $\frac{720°}{6}$ = | **120°** |
| *n*-gon | *n* | *n* – 2 | (*n* – 2)180° = | (*n* – 2)180° | $\frac{(n-2)180°}{n}$ = | $\frac{(n-2)180°}{n}$ |

b. Use the pattern in the table to find the interior angle measure of a regular decagon (10 sides).

**Sample Answer:** $\frac{(10-2)180°}{10} = 144°$

**Teacher Tip:** Use this as a "teachable moment" when completing the table for the *n*-gon.

**TI-Nspire Navigator Opportunity: *Quick Poll***
**See Note 3 at the end of this lesson.**

4. The interior angle sum can be calculated using Joshua's expression (*n* – 2)180, or 180*n* – 360 (as done by Madeline).

a. What is the relationship between these two expressions?

**Sample Answer:** The expressions (*n* – 2)180 and 180*n* – 360 are equivalent according to the commutative and distributive properties.

**Teacher Tip:** When discussing the expression (*n* – 2)180, consider identifying the need for grouping symbols around the expression (*n* – 2). You may wish to use the distributive property to show students that the expressions (*n* – 2)180 and *n* – 2(180) are not equivalent.

b. How can these expressions be modeled geometrically?

**Sample Answer:** The expression ($n$ – 2)180 can be modeled geometrically by drawing noncongruent segments from a single vertex in a polygon to its remaining vertices. The sum of the interior angles of these triangles is the sum of the interior angles of the polygon. The expression 180$n$ – 360 can be modeled geometrically by drawing congruent segments from the center of a polygon to each vertex. The sum of the interior angles of the polygon is the sum of the interior angles of these triangles minus the sum of the central angles, which is always 360°.

**Teacher Tip:** Encourage students to refer to problems 2 and 3 in the .tns file.

5. An irregular polygon is not equiangular and equilateral. Can Madeline or Joshua's methods be used to determine the interior angle sum of an irregular polygon? Why or why not?

**Answer:** Both methods can be used to determine the interior angle sum of an irregular polygon because, for each method, the number of triangles drawn inside a polygon does not change when the polygon has different side lengths and angle measurements.

**Teacher Tip:** You may want to sketch this situation to verify the answer. You could also have students insert a new page and sketch the situation in the .tns file.

## Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:

- How to determine the central angle measure of a regular polygon.
- The relationship between the sum of the interior angles of a triangle and the sum of the interior angles of a regular and irregular polygon.
- How to apply geometric representations of the expressions ($n$ – 2)180 and 180$n$ – 360 to determine the measure of the interior angle of a regular polygon.

## TI-Nspire Navigator

**Note 1**

**Question 1, *Quick Poll*:**

Have students send in their response to 1c in an open response Quick Poll. Ask students to send in the measure of central angles for other polygons as needed for additional practice.

**Note 2**

**Question 2, *Quick Poll*:**

Send students the following open response Quick Poll.

What is the measure of one interior angle in a regular dodecagon?

**Answer:** 150°

**Note 3**

**Question 3, *Quick Poll*:**

Send students the following open response Quick Poll.

What is the sum of the measures of the interior angles in a regular octagon?

**Answer:** 1080°