Math Objectives

- Students will infer why the conditions $b > 0$ and $b \neq 1$ are necessary for the function to be exponential.
- Students will determine that for $b > 1$ the function is increasing and for $0 < b < 1$ the function is decreasing.
- Students will determine that the *y*-intercept is always $(0,1)$ and there is no *x*-intercept.
- Students will determine that for $b > 1$ the function approaches ∞ as *x* approaches ∞ and that for $0 < b < 1$ the function approaches ∞ as *x* approaches $-\infty$.
- Students will identify the domain as $(-\infty, \infty)$ and the range as $(0, \infty)$.
- Students will identify the equation of the function's horizontal asymptote as $y = 0$.
- Students will construct viable arguments & critique the reasoning of others (CCSS Mathematical Practice).

Vocabulary

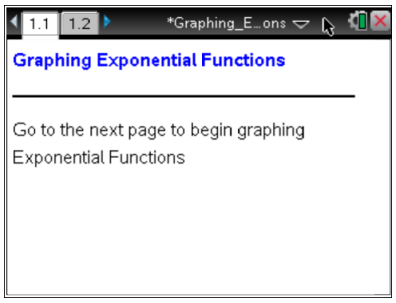
- exponential function
- end behavior
- intercepts
- domain and range
- asymptotes
- increasing and decreasing functions

About the Lesson

- Students will investigate the graphs of the family of exponential functions $f(x) = b^x$.
- As a result, students will:
  - Infer why the conditions $b > 0$ and $b \neq 1$ are necessary.
  - Determine how the value of *b* affects the increasing or decreasing behavior of the function.
  - Determine the *y*-intercept, domain, and range.
  - Describe the end behavior.
  - State the equation of the asymptote.

TI-Nspire™ Navigator™ System

- Use Live Presenter to demonstrate how to use sliders.
- Use Screen Capture to examine patterns that emerge.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Use a minimized slider

Tech Tips:

- Make sure the font size on your TI-Nspire handheld is set to Medium.
- You can hide the entry line by pressing ctrl G.

Lesson Materials:

Student Activity

Graphing_Exponential_Functions_Student.pdf

Graphing_Exponential_Functions_Student.doc

Optional Materials:

Graphing_Exponential_Functions_Create.doc

Graphing_Exponential_Functions_Create.pdf

TI-Nspire document

Graphing_Exponential_Function.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.

- Use Quick Poll to assess students' understanding throughout the activity.

Discussion Points and Possible Answers

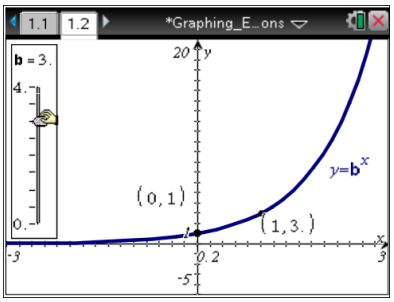
TI-Nspire Navigator Opportunity: Live Presenter
See Note 1 at the end of this lesson.

Teacher Tip: Students can either use the premade file or use the create activity to create the TI-Nspire document file first.

Move to page 1.2.

1. Explore several different *b*-values by dragging the slider.
   a. Set *b* = 1. Describe the graph.

   Answer: It is the horizontal line *y* = 1.



   b. By definition, for the exponential function $f(x) = b^x$, *b* cannot equal 1. What mathematical reason can you give for this restriction?

   Answer: When *b* = 1, the function becomes a linear relationship and is no longer an exponential function.

   c. Set *b* = 0. Describe the graph.

   Answer: It is a horizontal line for $x > 0$. It is the line $y = 0$.

   d. By definition, for the exponential function $f(x) = b^x$, *b* cannot equal 0. What mathematical reason can you give for this restriction?

   Answer: When *b* = 0, the function becomes a linear relationship and is no longer an exponential function.

2. Explore several different *b*-values by dragging the slider.
   a. For what *b*-values is the function increasing? Why is this true?

Answer: When $b > 1, f(x) = b^x$ is increasing because $b^n > b^m$ when $n > m$.

b. For what *b*-values is the function decreasing? Why is this true?

Answer: When $0 < b < 1, f(x) = b^x$ is decreasing because $b^n < b^m$ when $n > m$.

TI-Nspire Navigator Opportunity: Screen Capture
See Note 2 at the end of this lesson.

3. Explore several different *b*-values by dragging the slider.
   a. For each *b*-value, identify the *y*-intercept of the function. Interpret your results.

   Answer: The *y*-intercept is always $(0, 1)$ because $b^0 = 1$ for all *b.*

   b. When $b > 0$, why is there no *x*-intercept?

   Answer: There is no exponent for *b* that would result in an answer of 0.

   c. Another special point is when *x* = 1. Describe the general point regardless of the value of *b*. Explain your answer.

   Answer: When *x* = 1, the point will always be (1, *b*) because the base raised to the power of 1 will be the value of *b*.

TI-Nspire Navigator Opportunity: Screen Capture:
See Note 3 at the end of this lesson.

Teacher Tip: If students have not encountered asymptotes before, there may need to be some discussion before they answer the next questions.

4. Drag the slider to explore several different *b*-values where $b > 1$.
   a. What does $f(x)$ approach as *x* approaches ∞? Explain.

   Answer: ∞ ; As you choose larger and larger positive exponents for *b*, the result will be greater and greater.

   b. What does $f(x)$ approach as *x* approaches $-\infty$? Explain.

Answer: 0; As you choose smaller and smaller negative exponents for *b*, the result will get closer and closer to 0 without ever reaching 0.

c. What is the equation of the horizontal asymptote?

Answer: $y = 0$

5. Drag the slider to explore several different *b*-values where $0 < b < 1$.
   a. What does $f(x)$ approach as *x* approaches $-\infty$? Explain.

Answer: ∞ ; As you choose smaller and smaller negative exponents for *b*, the result will get larger and larger.

b. What does $f(x)$ approach as *x* approaches ∞? Explain.

Answer: 0; As you choose larger and larger exponents for *b*, the result will get closer and closer to 0 without ever reaching 0.

c. What is the equation of the horizontal asymptote?

Answer: $y = 0$

6. Find the domain and range for the family of exponential functions $f(x) = b^x$ where $b > 0$ and $b \neq 1$.

Answer: The domain is $(-\infty, \infty)$ and the range is $(0, \infty)$.

TI-Nspire Navigator Opportunity: Quick Poll
See Note 5 at the end of this lesson.

7. Wade believes the function $f(x) = b^x$ will eventually intersect the *x*-axis. Is he correct? Why or why not?

Answer: Wade is incorrect. The function will never cross the *x*-axis. It will only approach the *x*-axis.

TI-Nspire Navigator Opportunity: Quick Poll
See Note 6 at the end of this lesson.

8. Eric believes that for $b > 1$ the function $f(x) = b^x$ increases on only one side of the *y*-axis. Is he correct? Why or why not?

   Answer: Eric is incorrect. The function increases more quickly on one side of the *y*-axis, but it increases over the entire domain.

TI-Nspire Navigator Opportunity: Live Presenter
See Note 7 at the end of this lesson.

Extension: Students have been considering functions of the form $f(x) = b^x$, where $b > 0$, $b \neq 1$. Ask students to make conjectures about the characteristics and behavior of the function when $b < 0$.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand that for graphs of $f(x) = b^x$:

- The conditions $b > 0$ and $b \neq 1$ are necessary.
- For $b > 1$ the function is increasing, and for $0 < b < 1$ the function is decreasing.
- The *y*-intercept is always (0, 1) and there is never an *x*-intercept.
- For $b > 1$ the function approaches ∞ as *x* approaches ∞.
- For $0 < b < 1$ the function approaches ∞ as *x* approaches $-\infty$.
- The domain is $(-\infty, \infty)$ and the range is $(0, \infty)$.
- The function has a horizontal asymptote of $y = 0$.

TI-Nspire Navigator

Note 1

Before the lesson, *Live Presenter*: You may want to demonstrate or have a student use Live Presenter to show how to change the *b*-values by dragging the slider.

Note 2

Question 2a and 2b, *Screen Capture*: Take a Screen Capture of page 1.2 when students are on different *b*-values. As a class, discuss the various cases that occur.

Note 3

Question 3a and 3b, *Screen Capture*: Take a Screen Capture of page 1.2 when students are on different *b*-values. As a class, discuss the various cases that occur.

Note 4

Question 4a – 4f, *Screen Capture*: Take a Screen Capture of page 1.2 when students are on different *b*-values. As a class, discuss the various cases that occur.

Note 5

Question 6, *Quick Poll* (*Open Response*): Send two Open Response Quick Polls, asking students to submit their domain and range. If students struggle to identify the domain and range, consider taking a Screen Capture and discussing how, for all graphs, the possible *x*- and .*y*-values are similar.

Note 6

Question 7, *Quick Poll* (*Open Response*): Send an Open Response Quick Poll, asking students to submit their answer to Question 7.

Note 7

Question 8, *Quick Poll* (*Open Response*): Send an Open Response Quick Poll, asking students to submit their answer to Question 8.