

Binomial Distribution Graphs

Student Activity Sheet

7 8 9 10 11 12



Introduction

This is the first activity in the *Probability and Statistics with Pizzazz* series.

This activity focuses on the binomial probability mass function and its graph.

The second exploration in the series focuses on graphs of probability density functions for continuous random variables, including the normal and standard normal distributions.

A probability function may be represented as a formula (usually an equation), a table or a graph. In this activity, you will use interactive graphs in TI-Nspire CAS to explore the effect that changing the value of a parameter (a number describing some characteristic of the distribution) has on the graph of that probability function.

Exploration 1. Binomial Graphs

1.1 Table of the probability function for $X \sim \text{Bi}(10, p)$

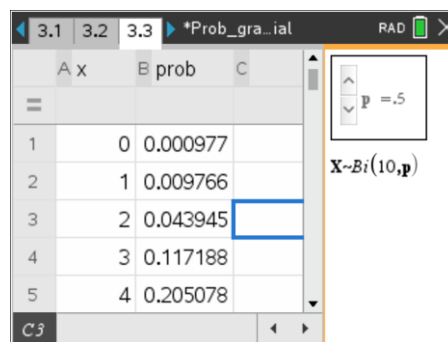
Consider a video game in which the player gets 10 shots at a target. Each shot is a binomial trial in which the probability of success (hitting the target) in a single trial, p , depends on the level of difficulty chosen by the player (that is, there are differing values of p).

If the random variable X is the number of successes in the 10 trials, then $X \sim \text{Bi}(10, p)$ and

$Pr(X = x) = {}^n C_x \cdot p^x \cdot (1 - p)^{n-x}$, where $x = \{0, 1, 2, \dots, 10\}$ and $0 \leq p \leq 1$.

Please refer to the TI-Nspire document 'Prob_graphs_binomial'

Open the TI-Nspire document. Navigate to page 3.3. On this page, the probability of observing 0, 1, 2, ..., 10 successes in 10 trials is computed in the second column of the spreadsheet. Use the slider on the right to change the value of p .



A	x	B	prob	C
1	0	0.000977		
2	1	0.009766		
3	2	0.043945		
4	3	0.117188		
5	4	0.205078		

Question 1

When $p = 0.5$, describe any patterns that you observe when considering pairs of probabilities such as $Pr(X = 0)$ and $Pr(X = 10)$, $Pr(X = 1)$ and $Pr(X = 9)$. Explain why this pattern occurs.

Question 2

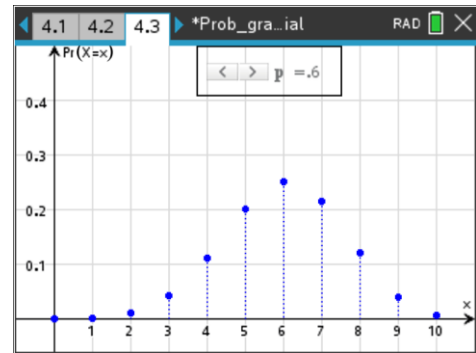
For the 10 trials, when $p = 0.5$,

- What is the 'most likely' outcome?
- On average, how many successes would you expect to observe from 10 shots at the target?

1.2 Graph of the probability function for $X \sim \text{Bi}(10, p)$

Navigate to page 4.3 of the TI-Nspire document. The values from the probability table have been plotted in the 'Graphs' application, and the slider controls the value of p .

Consider the graphs for 10 trials and $p = 0.25$, $p = 0.5$ and $p = 0.75$ to answer Questions 3 to 5 below.



Question 3

Adjust the graph on page 4.3 for $n = 10$, $p = 0.25$.

- Describe the shape and any other interesting features of the graph.
- What is the 'most likely' number of successes from 10 trials?
- What are the number of successes that are unlikely to be observed?

Question 4

Adjust the graph on page 4.3 for $n = 10$, $p = 0.5$.

- Describe the shape and any other interesting features of the graph.
- What is the 'most likely' number of successes from 10 trials?
- What are the number of successes that are unlikely to be observed?

Question 5

Adjust the graph on page 4.3 for $n = 10$, $p = 0.75$.

- Describe the shape and any other interesting features of the graph.
- What is the 'most likely' number of successes from 10 trials?
- What are the number of successes that are unlikely to be observed?

1.3 Expectation, $E(X)$, of the binomial distribution

Navigate to page 5.2 of the TI-Nspire document. On the right window, click any cell in the spreadsheet and right-arrow to column C, labelled xp . In this column, the product of the cells a1 and b1 is calculated in cell c1 (i.e. $x \cdot \text{prob}$), and this is repeated for each value of x and its associated probability.

	E prob	xp
1	0.000...	0.
2	0.000...	0.000024
3	0.000...	0.000507
4	0.001...	0.004947
5	0.007...	0.02968

$E(X) = \text{sum}(x \cdot p(x))$
 $\text{sum}(xp) = 9.$

The definition of expectation for a discrete random variable, X , is

$$E(X) = \sum x \cdot \Pr(X = x), \text{ which can also be written } E(X) = \sum x \cdot p(x)$$

On the left window of page 5.2, the Maths Box under the sliders calculates the product of the slider values, $n \times p$. The bottom Maths Box calculates the expected value, $E(X)$, by finding the sum of column C, which is labelled xp . That is, $\text{sum}(xp) = \sum x \cdot p(x) = E(X)$.

Question 6

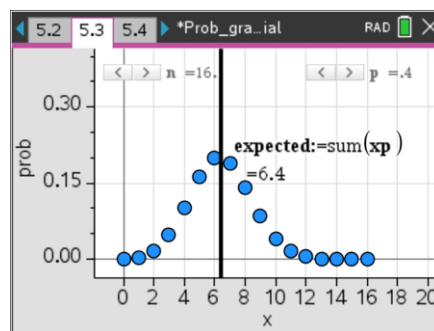
On page 5.2, evaluate the Maths Boxes 'sum(xp)' and 'n·p' (click the box and pressing <enter>).

- a. Complete the table on the right by adjusting the slider values and reading the value of $E(X)$ from the output of the 'sum(xp)' Maths Box.

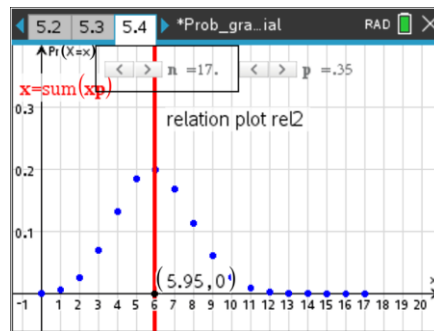
n	p	$n \times p$	$E(X)$
10	0.65		
20	0.25		
15	0.8		
12	0.35		
7	0.75		

- b. What do you observe about the relationship between n, p and $E(X)$ for these cases. Chose more values of n and p and note whether the relationship holds for any chosen values of n and p .

Page 5.3 displays graph of the binomial distribution in the 'Data & Statistics' application, with sliders controlling the values of n and p . The vertical line plots the expected value by calculating 'sum(xp)'. Click the vertical line to display the expected value.



Page 5.4 also displays the graph of the binomial distribution in the 'Graphs' application, with sliders controlling the values of n and p . The red vertical line plots the expected value by calculating 'sum(xp)'. The expected value is shown by the x-coordinate of the intersection of the red line and the horizontal axis.



Question 5

Use the graphs to observe the any relationships between shape (skew) of the graph and the expected value of the distribution. Comment on your observations.

Shape	Comment on location of the expected value, $E(X)$
Symmetrical	
Positively skewed	
Negatively skewed	