Infestation to Extermination

ID: 11856

Time Required
40 minutes

Activity Overview

In this activity, students will investigate exponential growth and decay through the situation of infestation and extermination. Students review foundational understanding and learn the process of solving separable differential equations. Application questions are solved numerically and graphically. As an extension, students explore other separable differential equations.

Topic: Exponential Growth & Decay

- *Solving separable differential equations*
- *Differentiating between exponential functions for growth and decay*

Teacher Preparation and Notes

- *The use of the accompanying handout is optional. Students can write their responses directly into the TI-Nspire handheld and/or on the worksheet. On self-check questions, after answering the question, students can press* menu *and select* ***Check Answer*** *(or* ctrl *+ ▲).*
- *The last section of this activity goes beyond exponential growth and decay. The CAS utility of* ***deSolve*** *allows students to explore the solution to differential equations.*
- *After finishing this activity students should be better equipped for AP* exam questions like 2006BC form B #5a, 2006AB form B #5c, and multiple-choice questions like 1998AB #21and #84.*
- *Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.*
- ***To download the student and solution TI-Nspire™ documents (.tns files) and student worksheet, go to education.ti.com/exchange and enter "11856" in the keyword search box.***

Associated Materials

- *InfestationToExtermination_Student.doc*
- *InfestationToExtermination.tns*
- *InfestationToExtermination_Soln.tns*

Suggested Related Activities

- *Exponential Growth and Decay (TI-Nspire technology) — 12359*
- *Slope Fields Introduction (TI-Nspire technology) — 16097*
- *Differential Equations (TI-Nspire CAS technology) — 8998*

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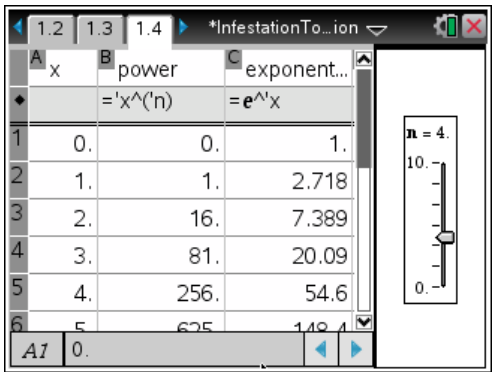
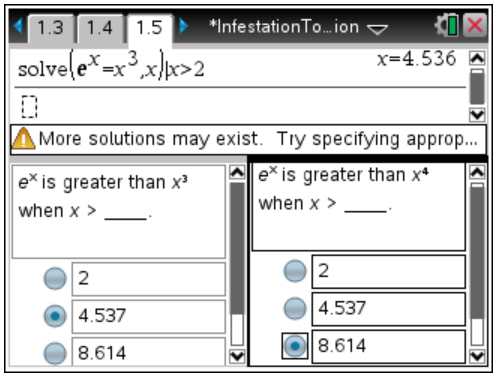
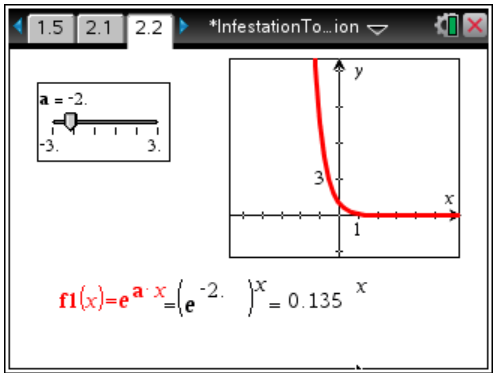
Part 1 – Warm Up

TI-Nspire Navigator Opportunity: *Class Capture*

See Note 1 at the end of this lesson.







Students review foundational understanding and are asked questions that will help them make connections between what they know and what they need to know to understand the process of solving separable differential equations.

Exponential functions and power functions are compared numerically using a spreadsheet. Students can use this spreadsheet or CAS to answer the questions on page 1.5.

For further thought and discussion:

- When n is 7, how big does x have to be so that e^x is greater than x^n? *When x is 22, $e^x > x^7$.*
- For larger values of n, which function increases at a faster rate? $y = e^x$
- What is $\lim_{x\to\infty} \frac{x^{25}}{e^x}$? *zero*

Using a dynamic slider bar, e^{ax} and b^x are graphically and algebraically related. Multiple-choice self-check questions enable students to get immediate feedback to strengthen their understanding of the difference between exponential growth and decay.

The final question in Part 1 helps students see how to solve for the constant k. Ask students to discuss with their neighbor the error in the other choices. *Answer: In b, instead of x = 4 and y = 9, those were mixed up. For c, log properties are improperly implemented, i.e. the 3 is not raised to the power of 4k.*

Student Solutions

1. $y = e^x$ is always increasing
2. $x > 4.537$ *
3. $x > 8.614$ *
4. For growth $a > 0$ and $b > 1$
5. For decay $a < 0$ and $0 < b < 1$
6. $k = \frac{\ln(3)}{4}$

*Note that the answers displayed at the top of page 1.5 of the solution document, InfestationToExtermination_Soln.tns, are rounded to three decimal places.

TI-Nspire Navigator Opportunity: *Quick Poll*

See Note 2 at the end of this lesson.

Part 2 – Infestation (Exponential Growth)

In Part 2, students examine the example of infestation of bugs in order to be introduced to the process of solving a separable differential equation.

The worksheet will encourage students to take note of the steps so they can repeat it on their own for Part 3.

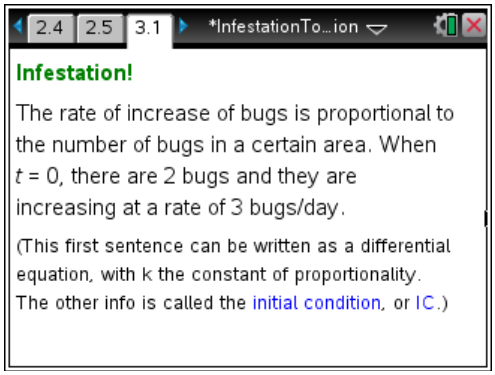
Multiple-choice questions lead students through the steps.

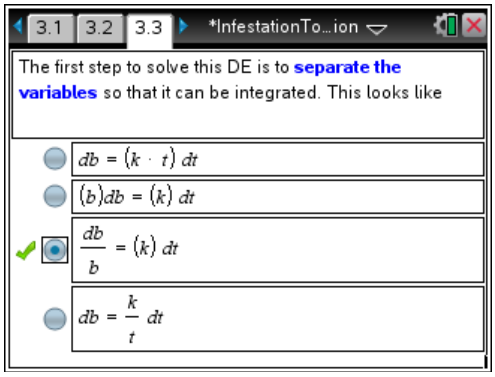
On page 3.7, point out or ask about the general solution. *It is the solution that has the constants in it. The initial conditions have not been applied.*

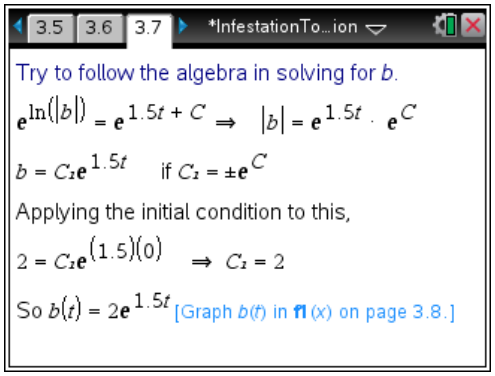
The step that gives students the most difficulty is the getting rid of the absolute value by defining the constant to be a value such that b is positive.

Students can answer application questions graphically using **Trace** or use the *Calculator* pages given.

A note about using **Graph Trace**: Press **MENU > Trace > Graph Trace**. Entering the x-value of interest, for example $x = 2$ in Question 6, will "jump" the trace to that point. When tracing on a function, a point can be "dropped" by pressing enter. Another option is to select **MENU > Geometry > Points & Lines > Point On**. Then, click the graph of the function to "drop" a point. The x- or y-value can be edited by clicking that value twice, entering a new value, and pressing enter.







TI-Nspire Navigator Opportunity: *Quick Poll*

See Note 3 at the end of this lesson.

Student Solutions

1. $\frac{db}{dt} = kb$

2. $\frac{db}{b} = k \cdot dt$

3. $\ln|b| = kt + C$

4. The "+ *C*" on the right includes of "+ *C*" on the left.

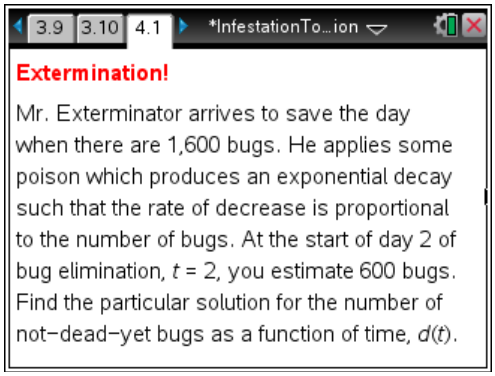
5. $3 = k(2)$, so $k = 1.5$; $b(t) = 2e^{1.5t}$

6. $b(2) = 40$, $b(3) = 180$

7. Just over 4.25 days. Solve $1200 = 2e^{1.5t}$ for *t*.

Part 3 – Extermination (Exponential Decay)

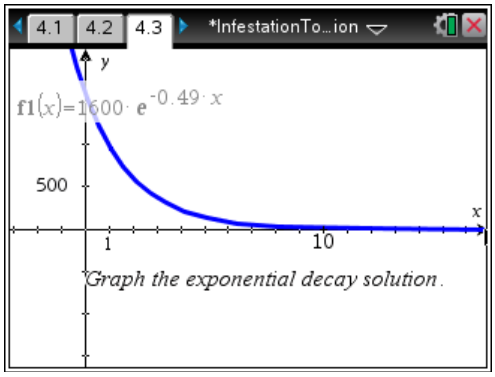
Students apply what was learned from "Infestation" to "Extermination." Students see the similarity between exponential growth and decay. They graphically, numerically, or with the aid of CAS, solve several application questions.



For further thought and discussion:

- What would have to be a characteristic of the poison, so that exponential decay occurs during this time? *The poison must also block further reproduction at the previous exponential growth rate.*

Students may need help solving the first question and finding the constants from the initial condition.



Student Solutions

1. $d(t) = 1600e^{-0.49t}$

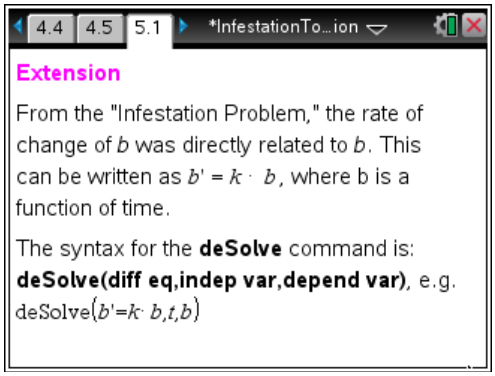
2. $d(10) = 12$

3. Yes, you need another application because there are still bugs left once the poison is gone.

4. Just over 15 days

Part 4 – Extension (CAS deSolve)

Part 4 introduces students to the use of **deSolve**. The syntax is demonstrated and students use the **Evaluate** tool in a *Notes* page to confirm their previous solutions. Other separable differential equations are explored. Through this, students may discover that differential equations are not all as intimidating as they may have thought. Graphs with sliders, like on page 5.3, can also be used to segue to slope fields.



Student Solutions

1. Circle $y^2 = -x^2 + C$

2. The constant *c1*, means C_1, *c2* means C_2, etc. AND, the constants are different.

TI-Nspire Navigator Opportunity: *Quick Poll*

See Note 4 at the end of this lesson.

TI-Nspire Navigator Opportunities

Note 1
Part 1, *Class Capture*
This would be a good place to use Class Capture to verify that students are able to correctly enter the values into the table and answer the questions.

Note 2
Part 1, *Quick Poll*
Use Quick Poll here or throughout the lesson to assess student understanding. The worksheet questions can be used as a guide for possible questions to ask.

Note 3
Part 2, *Quick Poll*
Use Quick Poll here or throughout the lesson to assess student understanding. The worksheet questions can be used as a guide for possible questions to ask.

Note 4
Part 3, *Quick Poll*
Use Quick Poll here or throughout the lesson to assess student understanding. The worksheet questions can be used as a guide for possible questions to ask.